Introductory examples of imprecise probability in environmental risk analysis

Ullrika Sahlin
Tuesday 16.00-17.30
Outline

• Uncertainty part I
• Introduction to environmental risk analysis
• Uncertainty part II
• Examples of imprecise probability
Uncertainty in environmental risk analysis

part I

Ullrika Sahlin August 2016
A possible view on unc in environmental risk analysis

- **Uncertainty** (epistemic uncertainty, lack of knowledge) – REDUCABLE
- **Variability** (aleatory uncertainty, stochasticity, inherent randomness) – NOT REDUCABLE
- All uncertainty is epistemic!
- A separation of variability is made to capture the dynamics of the system we are modelling!
• A **variable** is a quantity that takes multiple values in the real world

• A **parameter** is a quantity that has a single true value
H is true with Pr $\theta$

Case A:
H is a repeatable event

Case B:
H is a unique event

• Interpret $\theta$ under the two cases!
• Suggest ways to quantify $\theta$!
• Is there any difference between the two cases and, if so, why?
Knowledge underlying a risk analysis

![Diagram showing the components of knowledge in risk analysis: Expert knowledge, Theory, and Data.]

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Ullrika Sahlin
Multi-Criteria Decision Analysis

(1) Identify the problem (i.e., the decision to be made)
(2) Formulate objectives
(3) Develop management alternatives
(4) Estimate consequences associated with each alternative
(5) Evaluate trade-offs and select preferred alternatives
(6) Monitor and allow for learning

## Contaminated Sediment Management Decision

### Decision Criteria

- **Cost**
  - $/Cubic Yard

- **Public Acceptance**
  - Acres

- **Human Health**
  - # of complete human exposure pathways
  - Largest Cancer Risk calculated for any one pathway

- **Ecological Health**
  - # of complete ecological exposure pathways
  - Largest Ecological Hazard Quotient (HQ) calculated for any one pathway

### Table of Alternatives

<table>
<thead>
<tr>
<th></th>
<th>Cost</th>
<th>Public Acceptance</th>
<th>Human Health</th>
<th>Ecological Health</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$/CY</td>
<td>Acres</td>
<td>Pathways</td>
<td>Pathways</td>
</tr>
<tr>
<td>Choice A</td>
<td>20</td>
<td>1000</td>
<td>24</td>
<td>38</td>
</tr>
<tr>
<td>Choice B</td>
<td>40</td>
<td>200</td>
<td>18</td>
<td>23</td>
</tr>
<tr>
<td>Choice C</td>
<td>60</td>
<td>5</td>
<td>12</td>
<td>14</td>
</tr>
</tbody>
</table>

Unc in knowledge and values

Value ambiguity

Knowledge uncertainty

Hage et al (2010). Futures
Who’s uncertainty?

”Uncertainty is personal and temporal. The task of uncertainty analysis is to express the uncertainty of the assessors, at the time they conduct the assessment: there is no single ”true” uncertainty.”

”Uncertainty analysis should begin early in the assessment process and not be left to end.”

EFSA’s uncertainty guidance (draft 2016)
Uncertainty about causal relationships and in extreme events.

Uncertainty in values and preferences over decision alternatives.

Uncertainty about causal relationships and in extreme events

Beware of uncertainty taxonomies during the coming slides!
# A Taxonomy and Treatment of Uncertainty

Table 1. The various sources of epistemic and linguistic uncertainty with their most appropriate general treatments (refer to relevant section for references related to the suggested treatment).

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>General Treatments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Epistemic Uncertainty</strong></td>
<td></td>
</tr>
<tr>
<td>Measurement error</td>
<td>statistical techniques; intervals</td>
</tr>
<tr>
<td>Systematic error</td>
<td>recognize and remove bias</td>
</tr>
<tr>
<td>Natural variation</td>
<td>probability distributions; intervals</td>
</tr>
<tr>
<td>Inherent randomness</td>
<td>probability distributions</td>
</tr>
<tr>
<td>Model uncertainty</td>
<td>validation; revision of theory based on observation; analytic error estimation (for meta-models)</td>
</tr>
<tr>
<td>Subjective judgment</td>
<td>degrees of belief; imprecise probabilities</td>
</tr>
<tr>
<td><strong>Linguistic Uncertainty</strong></td>
<td></td>
</tr>
<tr>
<td>Numerical vagueness</td>
<td>sharp delineation; supervvaluations; fuzzy sets; intuitionistic, three-valued, fuzzy, paraconsistent and modal logics; rough sets</td>
</tr>
<tr>
<td>Nonnumerical vagueness</td>
<td>construct multidimensional measures then treat as for numerical vagueness</td>
</tr>
<tr>
<td>Context dependence</td>
<td>specify context</td>
</tr>
<tr>
<td>Ambiguity</td>
<td>clarify meaning</td>
</tr>
<tr>
<td>Indeterminacy in theoretical terms</td>
<td>make decision about future usage of term when need arises</td>
</tr>
<tr>
<td>Underspecificity</td>
<td>provide narrowest bounds; specify all available data</td>
</tr>
</tbody>
</table>
Fig. 1. A classification system for incertitude (Stirling and Gee\textsuperscript{(16)}).

### Fig. 1. A suggested taxonomy of uncertainties.

<table>
<thead>
<tr>
<th>Determinism</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Context</strong></td>
<td>A clear enough future</td>
<td>Alternate futures (with probabilities)</td>
<td>A multiplicity of plausible futures</td>
<td>Unknown future</td>
</tr>
<tr>
<td><strong>System model</strong></td>
<td>A single system model</td>
<td>A single system model with a probabilistic parameterization</td>
<td>Several system models, with different structures</td>
<td>Unknown system model; know we don’t know</td>
</tr>
<tr>
<td><strong>System outcomes</strong></td>
<td>A point estimate and confidence interval for each outcome</td>
<td>Several sets of point estimates and confidence intervals for the outcomes, with a probability attached to each set</td>
<td>A known range of outcomes</td>
<td>Unknown outcomes; know we don’t know</td>
</tr>
<tr>
<td><strong>Weights on outcomes</strong></td>
<td>A single estimate of the weights</td>
<td>Several sets of weights, with a probability attached to each set</td>
<td>A known range of weights</td>
<td>Unknown weights; know we don’t know</td>
</tr>
</tbody>
</table>
1. Future events

2. Parameters

3. Model structure

4. Known unknowns - "Low confidence"

5. Unknown unknowns "Black swans" svanar"

Spiegelhalter and Riesch (2011). Don’t know, can’t know: embracing deeper uncertainties when analysing risks. Phil. Trans. R. Soc. A
Unc VI

- **Type:** Substantive, Contextual, Procedural
- **Location:** Problem framing, Knowledge production, Communication and use
- **Source:** Lack of knowledge, Variability, Expert subjectivity, Communication patterns
- **Nature:** Epistemological, regulatory, socio-economic, transparency, fairness, inclusiveness, operational, competence, value-ladenness, linguistic, technical, methodological, preciseness, legitimacy

Fig. 1. Representations of several locations and sources of “problematic knowledge” in the literature.

Maxim and van der Sluijs (2011)
Environmental risk analysis – an introduction

Ullrika Sahlin August 2016
https://www.weforum.org/reports/the-global-risks-report-2016/
Figure 1.1: The Changing Global Risks Landscape 2015–2016: The 10 Most Changing Global Risks
Chemical use

• Chemical safety!
  – Protect species from high concentrations of dangerous chemicals

• Endpoints: Genes, individual organisms, populations, meta-populations, species communities
The exposure and effect paradigm

Endpoints
Stessors

• Chemicals
• Habitat loss
• Hunting pressure
• Natural hazards
  – e.g. storms or flooding
• Biological stessors
  – e.g. non-indigenous species or new diseases
• Changes in abiotic factors
  – e.g. climate change
  – Landuse change
Chemical hazard assessment

Species community

Species Toxicity

Proportion Affected Species

Small data!

EC50

Hazardous concentration
Habitat loss

- Conserve habitats to protect species from local or global extinction
- Restore habitats or build spreading corridors
- Risk assessed by Population Viability Analysis (PVA)
  - one or several populations
  - single or multiple species
The Population Viability Analysis paradigm

• Predict risk of extinction
• Consider population dynamics
• Include relevant links between environment and the dynamic of a population
• Include stochastic noise in population dynamics and environment
• Ecosystem based approach – consider also indirect effects via other species in the system
The IUCN Red List of Threatened Species

• Classification of risk status of species

Credal classification?
Over fishing

• Intensive fishing may cause crash of fish populations and future fishery
• Risk analysis e.g. PVA to find suitable levels of fishing intensity
• Spatial planning to identify areas protected from fishing

Robust strategies for Partially Observable Markov Decision Process
A fishy risk analysis

- First order multivariate autoregressive model MAR(1)
- Maximum likelihood using Kalman Filters
- Data from 1974-2004

Small data

Figure 2. Model validation by means of fitting and hindcasting the historical stock dynamics of (a,b) Baltic cod, (c,d) sprat and (e,f) herring. The left column shows the fit of the BALMAR food-web model (Lindgren et al. 2009), where SSB levels (black) accurately represent the observed dynamics (circles) of cod, sprat and herring from 1977 to 2004. (The degree of explained variance is: (a) 0.95; (c) 0.89 and (e) 0.98). The right column demonstrates hindcast SSB levels (black), where the historical stock dynamics were simulated based only on the starting biomasses (i.e. in 1977) as initial conditions. Grey lines are upper and lower 95% prediction intervals.
Forecasting under climate change
Figure 3. Future climate and management scenarios and a 95% probability distribution of Baltic cod SSB. (a) A ‘control scenario’ where climate (SST and salinity) and fishing mortalities ($F$) fluctuate at mean 1974–2004 levels. Hindcasted simulations from 1977 to 2007 (i.e. based on the observed climate and $F$ levels for these years) are compared with observed SSB (yellow circles) to validate the predictive accuracy of the model. (b) A predicted increase in mean SST by 3.5°C and decrease in mean salinity by 4.8 psu combined with mean $F$ levels. (c) As in (b) but with $F$ reduced to the previously recommended precautionary reference levels ($F_{pa}$). (d) Exploitation at $F_{pa}$ but with a predicted decrease in salinity by only 0.8 psu. Solid horizontal lines mark the recommended ecological levels of Baltic cod, the precautionary stock level, $B_{pa}$ (green) and limiting stock level, $B_{lim}$ (red). (Note that the use of these biomass reference points is currently being re-evaluated). Black contour lines show the 90 and 95% prediction intervals within which the cod stock dynamics of each replicated run fluctuates.
# Uncertainty in model structure

| Biological ensemble modeling to evaluate potential futures of living marine resources |

<table>
<thead>
<tr>
<th>Single-species models</th>
<th>Multispecies models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prey-independent predator</td>
</tr>
<tr>
<td></td>
<td>(2, 3)</td>
</tr>
<tr>
<td>S → C1</td>
<td>S → C2</td>
</tr>
<tr>
<td></td>
<td>No model available</td>
</tr>
<tr>
<td>T → C3</td>
<td>T → C4</td>
</tr>
<tr>
<td></td>
<td>(6)</td>
</tr>
<tr>
<td></td>
<td>RV</td>
</tr>
</tbody>
</table>

*Note: S represents stock recruitment function, T represents climate forcing, and C represents population biomass.*
Ensemble modelling

Bounds on forecasting by credal averaging

Ecological Applications
Volume 23, Issue 4, pages 742-754, 1 JUN 2013 DOI: 10.1890/12-0267.1
http://onlinelibrary.wiley.com/doi/10.1890/12-0267.1/full#1051-0761-23-4-742-f02
The DPSIR paradigm

Environmental impact assessments

Drivers → State

Pressures

State → Response

Impact

Responses → Drivers
A DPSIR example
The ecosystem service concept

ECOSYSTEM SERVICES

Supporting
■ NUTRIENT CYCLING
■ SOIL FORMATION
■ PRIMARY PRODUCTION
■ ...

Provisioning
■ FOOD
■ FRESH WATER
■ WOOD AND FIBER
■ FUEL
■ ...

Regulating
■ CLIMATE REGULATION
■ FLOOD REGULATION
■ DISEASE REGULATION
■ WATER PURIFICATION
■ ...

Cultural
■ AESTHETIC
■ SPIRITUAL
■ EDUCATIONAL
■ RECREATIONAL
■ ...

LIFE ON EARTH - BIODIVERSITY

CONSTITUENTS OF WELL-BEING

Security
■ PERSONAL SAFETY
■ SECURE RESOURCE ACCESS
■ SECURITY FROM DISASTERS

Basic material for good life
■ ADEQUATE LIVELIHOODS
■ SUFFICIENT NUTRITIOUS FOOD
■ SHELTER
■ ACCESS TO GOODS

Freedom of choice and action
■ OPPORTUNITY TO BE ABLE TO ACHIEVE WHAT AN INDIVIDUAL VALUES DOING AND BEING

Health
■ STRENGTH
■ FEELING WELL
■ ACCESS TO CLEAN AIR AND WATER

Good social relations
■ SOCIAL COHESION
■ MUTUAL RESPECT
■ ABILITY TO HELP OTHERS

Source: Millennium Ecosystem Assessment
Managing pollinator capital
The value of green stuff around your fields
Evaluating nonindigenous species management in a Bayesian networks derived relative risk framework for Padilla Bay, WA, USA
Regional relative risk assessment

- Unc from discretisation?
- Variability mixed with epistemic uncertainty
- No data generating process
- Precise conditional probability tables
Communication
Challenges to uncertainty

(i) Partial knowledge
(ii) Small data
(iii) Expert’s disagreement
(iv) No established theory

- Reliable and valid risk assessments
- Successful stakeholder interaction
Uncertainty in environmental risk analysis

part II

Ullrika Sahlin August 2016
A novel strategy for uncertainty management

Procedure to assess uncertainty

- Standardised procedures with accepted provision for uncertainty
- Case-specific assessments
  - Includes to develop or review a standardised procedure
- Emergency situations

Requires motivation!
Assessment components

Inputs → Propagation → Output

Most important for decision makers!
Main steps in uncertainty analysis

1. Identify and describe uncertainty qualitatively (source, cause, nature)
2. Assess individual sources of uncertainty
3. Assess the combined impact of all identified uncertainty in input taking account of dependencies
4. Assess the relative contribution of individual uncertainty to overall uncertainty
5. Document and report the uncertainty analysis
Assessment components

1. Identify sources to uncertainty

2. Assess individual sources to uncertainty

3. Assess combined impact of uncertainty on uncertainty in output

4. Assess relative contribution of sources of uncertainty
Methods

- Descriptive expression
- Ordinal scales
- Matrices
- NUSAP
- Uncertainty table
- Interval Analysis
- Expert knowledge elicitation

- Confidence Intervals
- The Bootstrap
- Bayesian Inference
- Probability Bounds Analysis
- Monte Carlo
- Conservative assumptions
- Sensitivity analysis

Step in the assessment

Types of assessment question
- Quantitative
- Categorical

Forms of uncertainty expression provided
- Descriptive
- Ordinal
- Range
- Range with probability
- Distribution
- Bound with probability
- Sensitivity of output to input uncertainty
Performance criteria on the method to assess uncertainty

• Evidence of current acceptance
• Expertise needed to conduct
• Time needed
• Theoretical basis
• Degree/ extent of subjectivity
• Method of propagation
• Treatment of uncertainty and variability
• Meaning of output
• Transparency and reproducibility
• Ease of understanding for non-specialist
Table 6: Criteria used in Table 5 for assessing performance of methods.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Evidence of current acceptance</th>
<th>Expertise needed to conduct</th>
<th>Time needed</th>
<th>Theoretical basis</th>
<th>Degree/extent of subjectivity</th>
<th>Method of propagation</th>
<th>Treatment of uncertainty and variability</th>
<th>Meaning of output</th>
<th>Transparency and reproducibility</th>
<th>Ease of understanding for non-specialist</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stronger characteristics</td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>International guidelines or standard scientific method</td>
<td>No specialist knowledge required</td>
<td>Hours</td>
<td>Well established, coherent basis for all aspects</td>
<td>Judgement used only to choose method of analysis</td>
<td>Calculation based on appropriate theory</td>
<td>Different types of uncert. &amp; var. quantified separately</td>
<td>Range and probability of alternative outcomes</td>
<td>All aspects of process and reasoning fully documented</td>
<td>All aspects fully understandable</td>
</tr>
<tr>
<td>B</td>
<td>EU level guidelines or widespread in practice</td>
<td>Can be used with guidelines or literature</td>
<td>Days</td>
<td>Most but not all aspects supported by theory</td>
<td>Combination of data and expert judgment</td>
<td>Formal expert judgment</td>
<td>Uncertainty and variability quantified separately</td>
<td>Range and relative possibility of outcomes</td>
<td>Most aspects of process and reasoning well documented</td>
<td>Outputs and most of process understandable</td>
</tr>
<tr>
<td>C</td>
<td>National guidelines, or well established in practice or literature</td>
<td>Training course needed</td>
<td>Weeks</td>
<td>Some aspects supported by theory</td>
<td>Expert judgment on defined qualitative scales</td>
<td>Informal expert judgment</td>
<td>Uncertainty and variability distinguished qualitatively</td>
<td>Range of outcomes but no weighting</td>
<td>Process well documented but limited explanation of reasoning</td>
<td>Outputs and principles of process understandable</td>
</tr>
<tr>
<td>D</td>
<td>Some publications and/or regulatory practice</td>
<td>Substantial expertise or experience needed</td>
<td>A few months</td>
<td>Limited theoretical basis</td>
<td>Expert judgment on defined ordinal scales</td>
<td>Calculation or matrices without theoretical basis</td>
<td>Quantitative measure of degree of uncertainty</td>
<td>Limited explanation of process and/or basis for conclusions</td>
<td>Outputs understandable but not process</td>
<td></td>
</tr>
<tr>
<td>Weaker characteristics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>Newly developed</td>
<td>Professional statistician needed</td>
<td>Many months</td>
<td>Pragmatic approach without theoretical basis</td>
<td>Verbal description, no defined scale</td>
<td>No propagation</td>
<td>No distinction between variability and uncertainty</td>
<td>Ordinal scale or narrative description for degree of uncertainty</td>
<td>No explanation of process or basis for conclusions</td>
<td>Process and outputs only understandable for specialists</td>
</tr>
</tbody>
</table>

Evaluate performance for some methods that you are familiar with!
Examples of imprecise probability

Ullrika Sahlin August 2016
Evidence: Observation of the species, \( E = \{0,1\} \)

Hypothesis: Species is present

Prior belief

\[ \theta \]

Detection probability

\[ dp \]

Evidence: Observation of the species, \( E = 0 \).

What is the probability that the species is still present?

What to do when experts disagree on \( \theta \)?

Quantify uncertainty in \( \theta \) when \( dp \) is an interval?
Daily intake exposure equation

\[ Dose = \frac{C \times IR \times EF}{bw} \]

C = concentration of chemical in medium (mg/l)
IR = intake/contact rate (l/day)
EF = exposure frequency
bw = body weight (mg)
Exposure data 1

C = [0.007, 3.30] x 10^{-3} \text{ mg/l}
IR = [4, 6] \text{ l/day}
EF = [45/365, 65/365]
bw = [4.514, 8.43] \text{ g}

• What is the worst case exposure?
Exposure data 2

\[ C = [0.007, 3.30] \times 10^{-3} \text{ mg/l} \]
\[ IR = [4, 6] \text{ l/day} \]
\[ EF \sim N(\ [50,60]\ /365, \ 5) \]

• Quantify uncertainty in a high exposure to an organism with \( bw = 5 \)?
• High exposure can be seen to occur in 1 day out of 100 (99th percentile).
Exposure data 3

C = \{0.001, 3.01, 0.74, 4.32, 2.9\} \times 10^{-3} \text{ mg/l}
IR = \{1.3, 4, 4.3, 5.9\} \text{ l/day}
EF \sim \text{N}(\frac{[50,60]}{365}, 5)

• C, IR, EF varies over time (variability)
• Quantify uncertainty in a high exposure to an organism with bw = 5?
• High exposure can be seen to occur in 1 day out of 100 (99th percentile).
Exposure data 4

\[ C = [0.007, 3.30] \times 10^{-3} \text{ mg/l} \]
\[ IR = [4, 6] \text{ l/day} \]
\[ EF > 55/365 \]
\[ bw = [4.514, 8.43] \text{ g} \]

- What is the worst case exposure?
Structural uncertainty

A

PLO

\[
Pfiesteria
\]

Fish kill

B

PLO

\[
Pfiesteria
\]

Fish kill

\textit{Pfiesteria} is a toxic algae. PLO are \textit{Pfiesteria}-like organisms.
Structural uncertainty

- \( \text{Pr}(\text{Pfiesteria}) = 0.03 \)
- \( \text{Pr}(\text{PLO} | \text{Pfiesteria}) = 1 \)
- \( \text{Pr}(\text{PLO}) = 0.35 \)
- \( \text{Pr}(\text{Fish kill} | \text{Pfiesteria}) = 1 \)
- \( \text{Pr}(\text{Fish kill}) = 0.073 \)
- \( \text{Pr}(\text{Pfiesteria} | \text{Fish kill}) = 0.38 \)

- What is the probability of Fish kills given that PLO is present under model A?

- \textit{Pfiesteria} were only present at fish kill sites and never elsewhere.

- What is the probability of Fish kills given the PLO is present under model B?
A prioritization problem

SETTING RELIABILITY BOUNDS ON HABITAT SUITABILITY INDICES
A prioritization problem

- Which patch should be prioritized for conservation?
- What if we need to eliminate a patch, which one should we take?
Spatial planning using PVA

• Two nature reserves $d$ distance apart
• $1/\beta = \text{mean disperal distance}$
• $U(\beta, u) = [(1 - u)\tilde{\beta}, (1 + u)\tilde{\beta}]$, where $0 < u < 1$ and $\tilde{\beta} = 0.05$ is the best guess
• $q = \text{the probability of persistence of the metapopulation under a long time horizon given by a meta-population model}$
• Optimal persistence when $\beta$ is precise is $R(\beta) = \max_d q(d)$
Spatial planning using PVA

• What distance should be between the reserves to make sure the persistence is acceptable, i.e.

$$\min_{\beta \in U(\tilde{\beta}, u)} R(\beta) \geq Q$$

$$q = \frac{e^{-\beta \cdot d} (2 \cdot p_e - 1) - (p_e - 1)[2 + (e^{-\alpha \cdot d} - 1) \cdot p_e]}{2} + \sqrt{\frac{4 \cdot (p_e - 1)[(e^{-\beta \cdot d} + p_e - 1)(p_e - 1) - e^{-\alpha \cdot d} \cdot p_e(p_e - e^{-\beta \cdot d} - 1)] + [2 - 3 \cdot p_e - e^{-\alpha \cdot d} \cdot p_e(p_e - 1) + p_e^2 + e^{-\beta \cdot d} (2 \cdot p_e - 1)]^2}{2}}$$

Info-gap analysis

• Find the distance $d$ which allows the most uncertainty in $1/\beta$ (i.e. the mean dispersal distance)

• $\hat{u}(d, Q) = \max \{ u: \min_{\beta \in U(\tilde{\beta}, u)} R(\beta) \geq Q \}$