Thursday 14:00-16:00

Part 9
Hands-on of imprecise simulation in engineering

by Edoardo Patelli and Roberto Rocchetta

The NAFEMS Challenge Problem

A Challenge Problem on Uncertainty Quantification & Value of Information

A mathematical model of a typical electronic device as represented by a R-L-C network will be provided along with different levels of uncertainty estimates around the input parameters.

The NAFEMS Challenge Problem

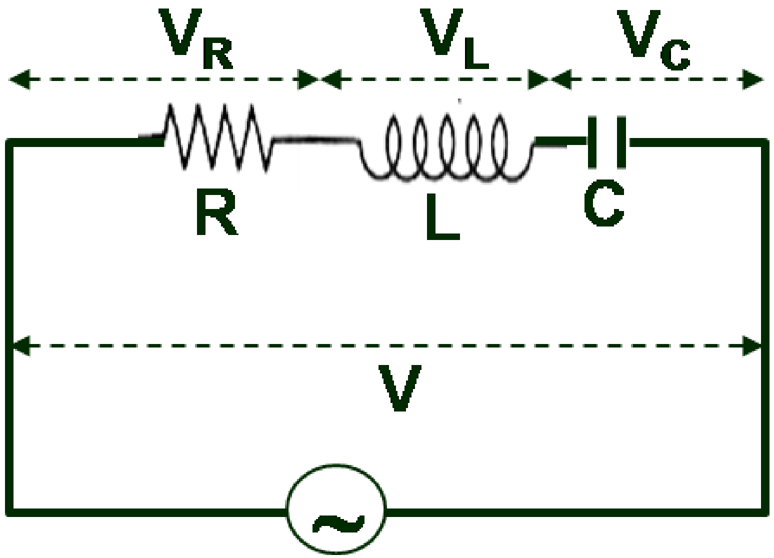
A Challenge Problem on Uncertainty Quantification & Value of Information

A mathematical model of a typical electronic device as represented by a R-L-C network will be provided along with different levels of uncertainty estimates around the input parameters.

The objective is to assess the reliability of the device based on a set of criteria and also to quantify the value of information.
The output response is sensitive to the model parameters that have different cases of value of information.

NAFEMS SWG

Problem statement



Typical electronic device represented by R-L-C network in series.

- **Input signal** Step voltage source for a short duration
- **Output response** Voltage at the capacitor (V_c)

Uncertainty estimates regarding the R, L, C values are available.
The challenge is to evaluate the reliability of the device using two criteria:

- Voltage at a particular time should be greater than a threshold
- Voltage rise time to be within a specified duration

The NAFEMS Challenge Problem

In a nutshell

Challenges:

- Deal with imprecision in the parameters data
- Assess quality of different information sources with respect to the reliability requirements

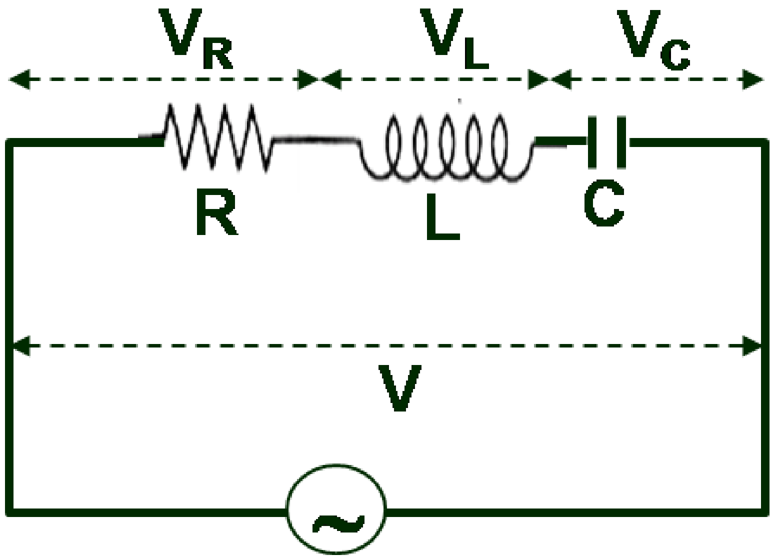
Resources:

- Analytical solution for the system output is provided
- Different information sources are available for the system parameters

References:

- `https://www.nafems.org/downloads/uq_value_of_information_challenge_problem_revised.pdf/`
- `https://www.nafems.org/downloads/stochastics_challenge_problem_nwc13.pptx/`

The Electrical Model



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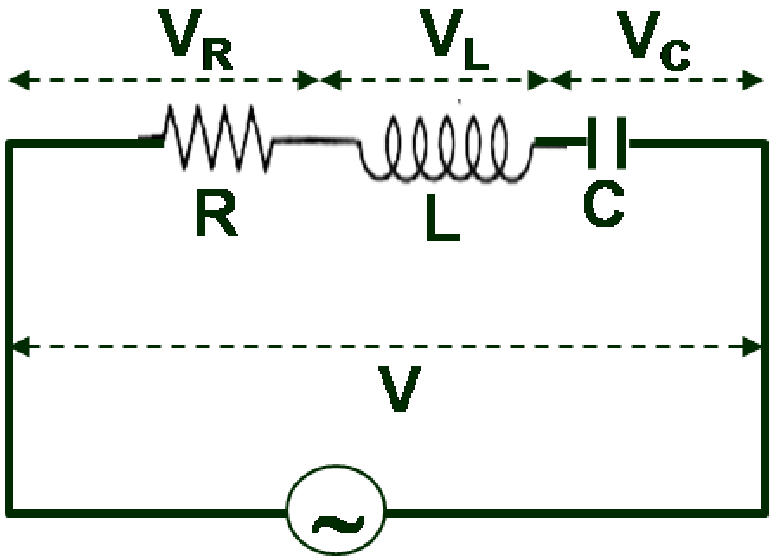
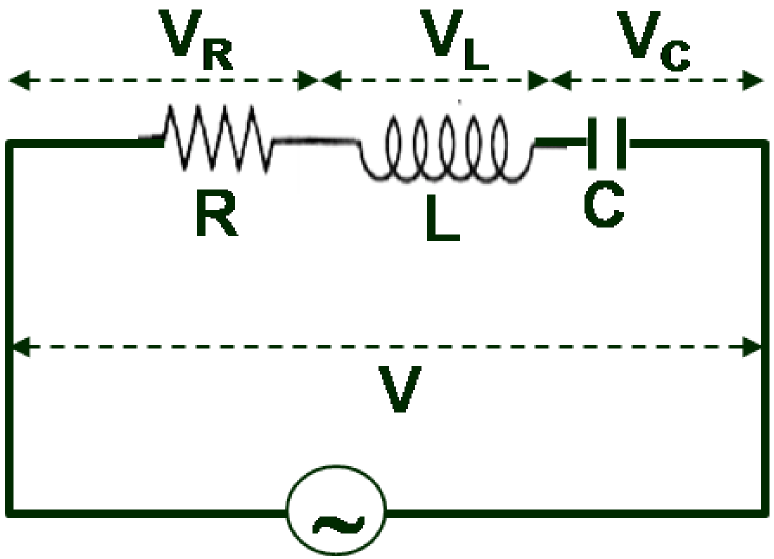
The R-L-C Model



The transfer function of the system is:

$$\frac{V_c(t)}{V} = \frac{\omega^2}{S^2 + \frac{R}{L}S + \omega^2} \qquad (40)$$

The R-L-C Model



The transfer function of the system is:

$$\frac{V_c(t)}{V} = \frac{\omega^2}{S^2 + \frac{R}{L}S + \omega^2} \tag{40}$$

Roots are computed as:

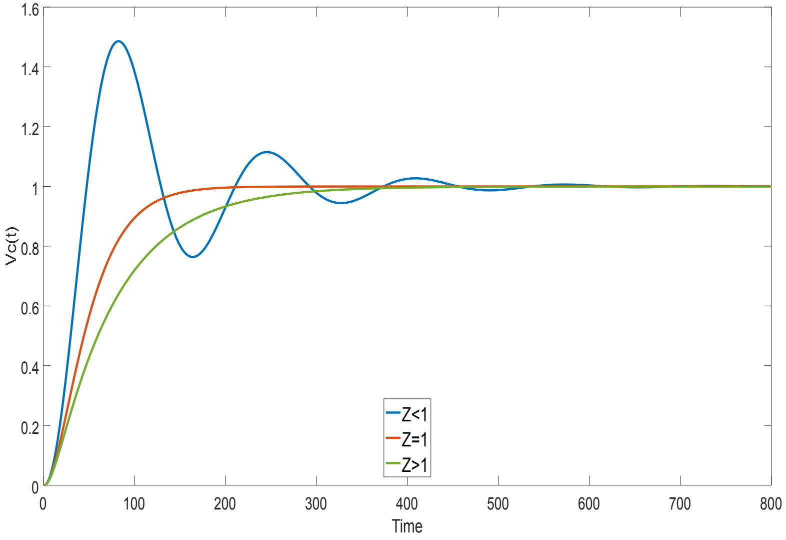
$$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega^2} \tag{41}$$

The system damping factor Z, parameter α and ω are determined as follow:

$$Z = \frac{\alpha}{\omega}; \quad \alpha = \frac{R}{2L}; \quad \omega = \frac{1}{\sqrt{LC}}; \tag{42}$$

The R-L-C Model

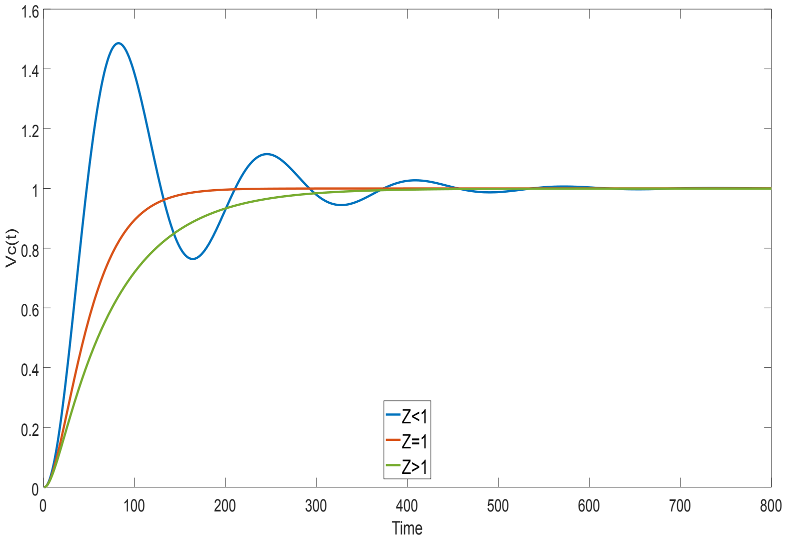
System response



Typical results: Under-damped, Critically-damped and Over-damped cases

The R-L-C Model

System response



Typical results: Under-damped, Critically-damped and Over-damped cases

Under-damped ($Z < 1$)

$$V_c(t) = V + (A_1 cos(\omega t) + A_2 sin(\omega t)) \exp^{-\alpha t} \tag{43}$$

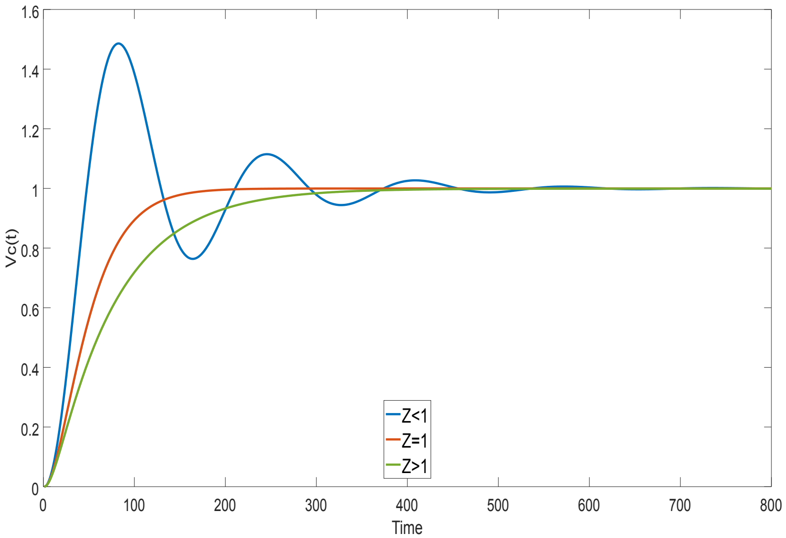
Critically-damped ($Z = 1$)

$$V_c(t) = V + (A_1 + A_2 t) \exp^{-\alpha t} \tag{44}$$

Over-damped ($Z > 1$)

$$V_c(t) = V + (A_1 \exp^{S_1 t} + A_2 \exp^{S_2 t}) \tag{45}$$

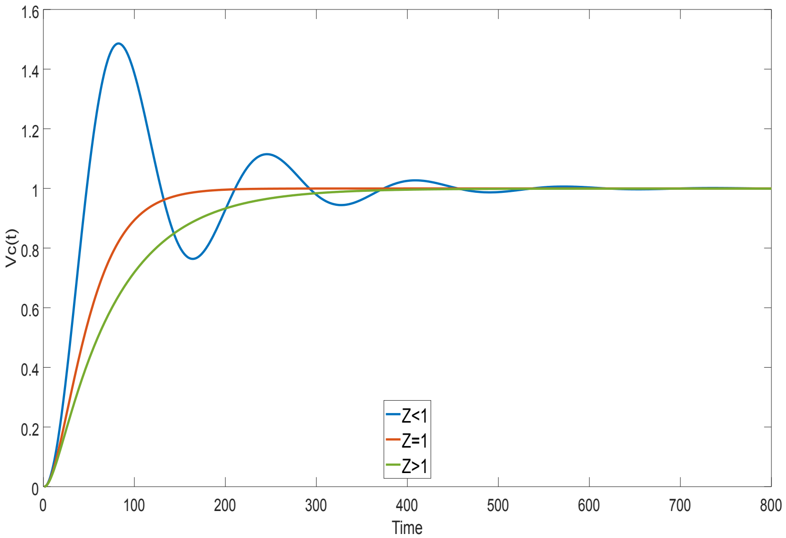
The R-L-C Model



Typical results: Under-damped, Critically-damped and Over-damped cases

For initial conditions $\frac{dV_c}{dt}\Big|_{t=0} = 0$ and $V_c(0) = 0$:

The R-L-C Model



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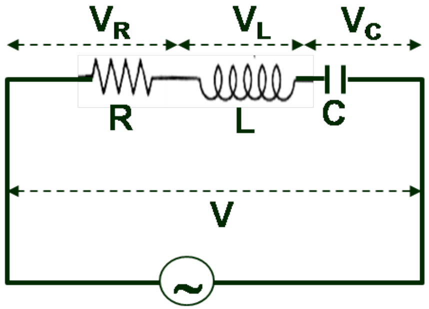
For initial conditions $\frac{dV_c}{dt}|_{t=0} = 0$ and $V_c(0) = 0$:

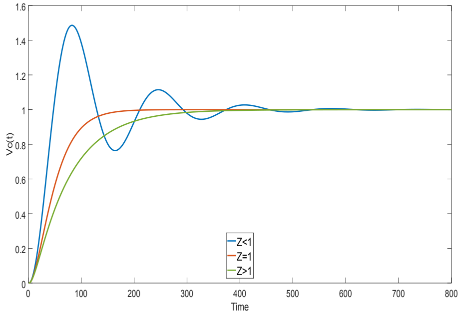
$$V_c(t) = V + (-cos(\omega t) - Z \cdot sin(\omega t))e^{-\alpha t} \;\; if \;\; Z < 1 \tag{46}$$

$$V_c(t) = V + (-1 - \alpha t)\, e^{-\alpha t} \;\; if \;\; Z = 1 \tag{47}$$

$$V_c(t) = V + \left(\frac{S_2}{S_1 - S_2} e^{S_1 t} + \frac{S_1}{S_2 - S_1} e^{S_2 t} \right) \;\; if \;\; Z > 1 \tag{48}$$

Reliability Requirements





Output of interest: The voltage at the capacitor

Reliability requirements on capacitance voltage (V_c), rise time (t_r) and damping factor (Z):

1. $V_c(t = 10ms) > 0.9\ V$
2. $t_r = t(V_c = 0.9V) \leq 8\ ms$
3. System should not oscillate ($Z > 1$)

Parameter Information

There are 4 cases, each affected by uncertainty/imprecision:

CASE-A	R [Ω]	L [mH]	C [μF]
Interval	[40,1000]	[1,10]	[1,10]

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CASE-B	R [Ω]	L [mH]	C [μF]
source 1	[40,1000]	[1,10]	[1,10]
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CASE-D	R [Ω]	L [mH]	C [μF]
Interval	[40,R_{U1}]	[1,L_{U1}]	[C_{L1},10]
Other info	$R_{U1} >$650	$L_{U1} >$6	$C_{L1} <$7
Nominal Val.	650	6	7

Hands-on of imprecise simulation in engineering

Outline

Possible solutions

Probabilistic approach

Parameter Characterisation

- Assign probability distribution to parameter values (e.g. uniform PDFs to intervals);
- Fit probability distribution using samples information (e.g. Kernels or Multivariate Gaussian);

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Uncertainty Quantification

- Propagate uncertainty using single loop Monte Carlo (MC);

Possible solutions

Imprecise probability approaches

Parameter Characterisation

- Dempster-Shafer structures (D-S)
- Probability-boxes (
- Fuzzy Variables

Possible solutions

Imprecise probability approaches

Parameter Characterisation

- Dempster-Shafer structures (D-S)
- Probability-boxes (
- Fuzzy Variables

Uncertainty Quantification

- Double Loop Monte Carlo
- D-S combination and propagation (i.e. Cartesian product of all focal elements + output mapping, min-max search);
- P-boxes propagation by α-cuts (i.e. focal element sampling + output mapping, min-max search);
- Robust Bayesian;
- Interval Analysis (e.g. min-max search)

Examples Imprecise probabilistic approach

Focal Elements Propagation, Remark:
In the procedure, m-dimensional interval input boxes are obtained (where m is number of focal elements sampled within each run). The output is then mapped by min-max searched within the m-dimensional box. This can be done in many ways, for instance:

1. Approximate by sampling (e.g. MC, LHC);
2. Optimization techniques (e.g. Genetic algorithm, quad. prog.);
3. Vertex method and Interval Arithmetic methods;
4. For monotonic systems responses w.r.t input parameters, the min-max are on the input domain boundaries;

Hands-on of imprecise simulation in engineering

Outline

Hands-on of imprecise simulation in engineering

Outline

General Remarks

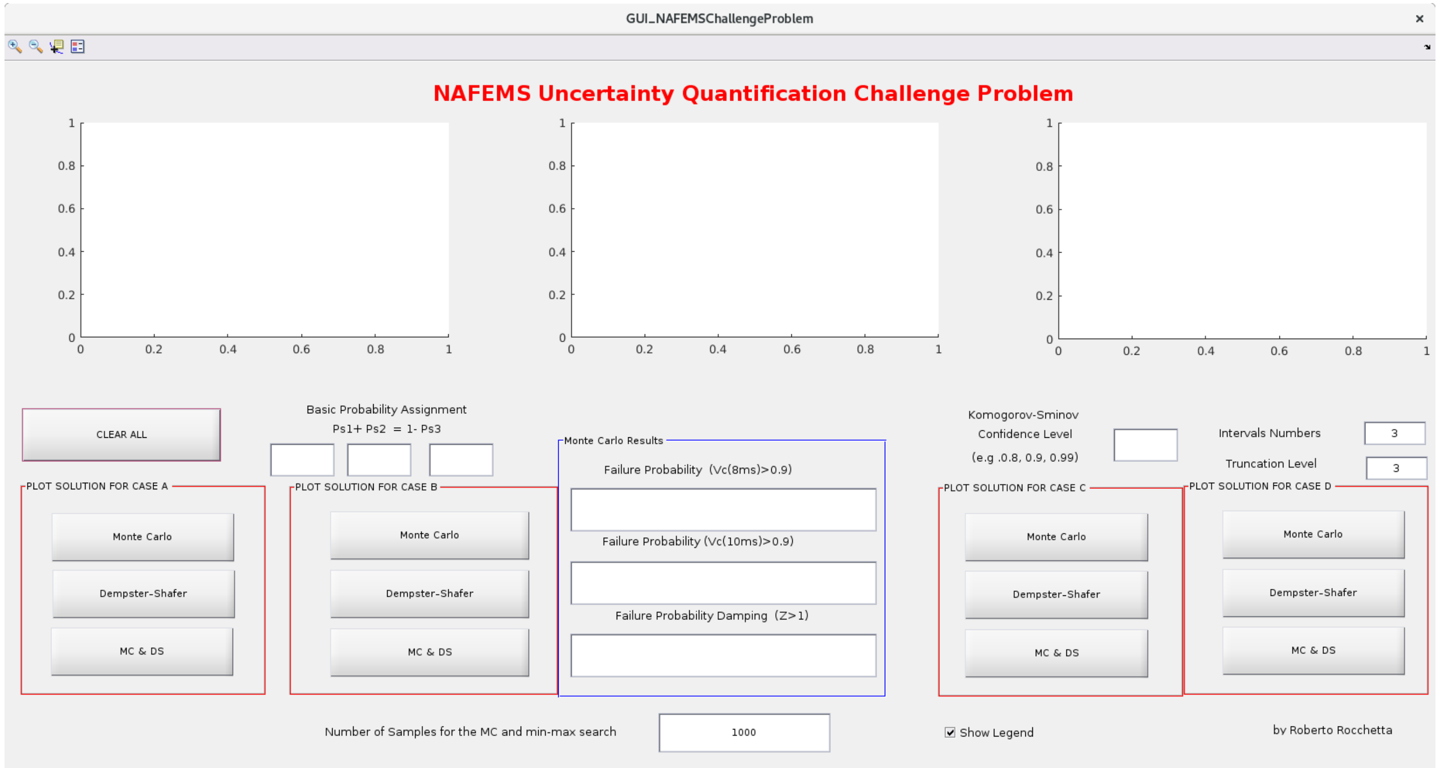
Tasks

- Evaluate the reliability of the R-L-C device using the given criteria
- Quantify the value of information in each case

- You can use a technique/strategy of your choice
- Each approach comes with it own limitations that need to be evaluated.

A reference solution is provided and accessible via a stand-alone app (shown in the next slide). It provides a possible solution and **not** the “true” answers to the problem (that are unknown).

Reference solution app



- Simple Stand alone application for the solution of the NAFEMS UQ Challenge problem
- Implement probabilistic approach (Monte Carlo) and D-S propagation

Hands-on of imprecise simulation in engineering

Outline

CASE-A

Intervals

Information: 3 intervals one for each system parameter.

CASE-A	R [Ω]	L [mH]	C [μF]
Interval	[40,1000]	[1,10]	[1,10]

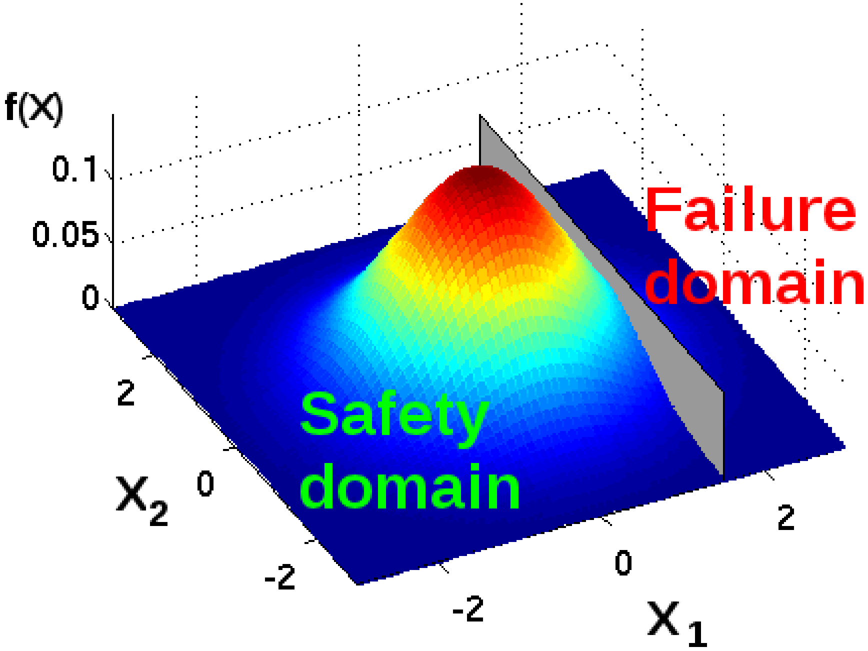
Provides reference solution obtained as follows:

Probabilistic approach:

- Maximum entropy principle, 3 uniform PDFs:
  $R \sim U(40, 1000)$, $L \sim U(1, 10)$, $C \sim U(1, 10)$;
- Propagate uncertainty via Monte Carlo simulation

Failure quantification

Bi-dimensional case



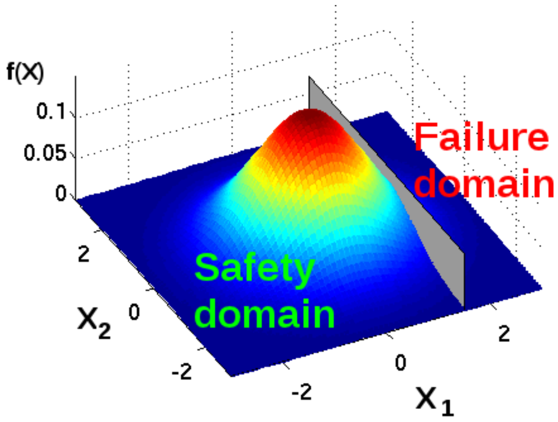
$$\int_{\mathcal{F}} f_{\mathbf{X}}(\mathbf{x})\ d\mathbf{x} = \int \mathbb{I}_{\mathcal{F}}(\mathbf{x})\ f_{\mathbf{X}}(\mathbf{x})\ d\mathbf{x}$$

where:

$$\mathbb{I}_{\mathcal{F}}(\mathbf{X}) = \begin{cases} 0 & \iff \quad \mathbf{X} \in \mathcal{S} \\ 1 & \iff \quad \mathbf{X} \in \mathcal{F} \end{cases}$$

Failure quantification

Monte Carlo simulation

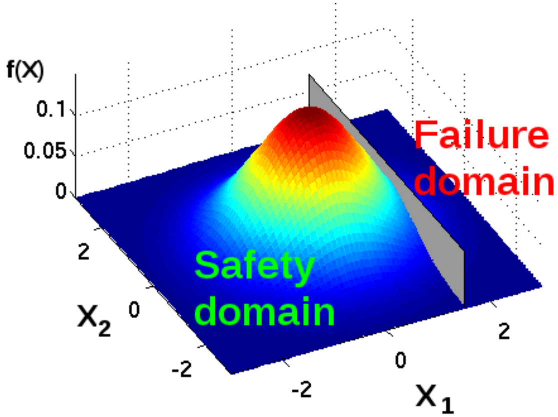


Evaluation ("dart" game):

- $f(\mathbf{x})d\mathbf{x}$ probability to hit a point
- $\mathbb{I}_{\mathcal{F}}(\mathbf{x})$ the prize

Failure quantification

Monte Carlo simulation



Evaluation ("dart" game):

- $f(\mathbf{x})d\mathbf{x}$ probability to hit a point
- $\mathbb{I}_{\mathcal{F}}(\mathbf{x})$ the prize

- Estimate: direct Monte Carlo simulation

$$P_f = \int \mathbb{I}_{\mathcal{F}}(\mathbf{x})\ f_{\mathbf{X}}(\mathbf{x})\ d\mathbf{x} \approx \frac{1}{N}\sum_{k=1}^{N} \mathbb{I}_{\mathcal{F}}(\mathbf{X}^{(k)})$$

- to meet specified accuracy: $N \propto \frac{1}{P_f}$

Estimate probability of failures : $\hat{P}_{Vc10}$, $\hat{P}_{tr}$, $\hat{P}_{Z}$

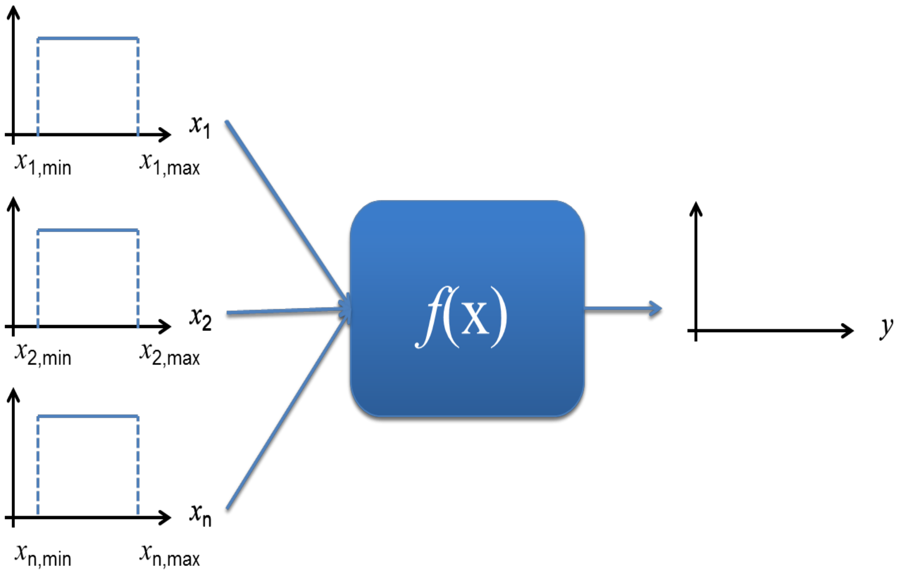
CASE-A

Intervals analysis

Information: 3 intervals one for each system parameter.

CASE-A	R [Ω]	L [mH]	C [μF]
Interval	[40,1000]	[1,10]	[1,10]

- Explore range of variation for $V_c(10ms)$, t_r and Z (min-max within input cuboid);



CASE-A

Intervals analysis

Information: 3 intervals one for each system parameter.

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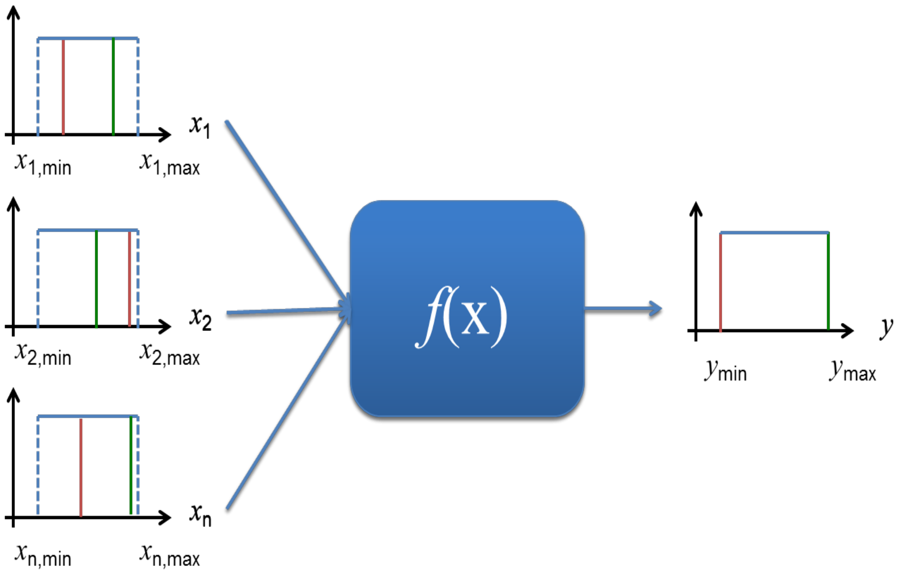
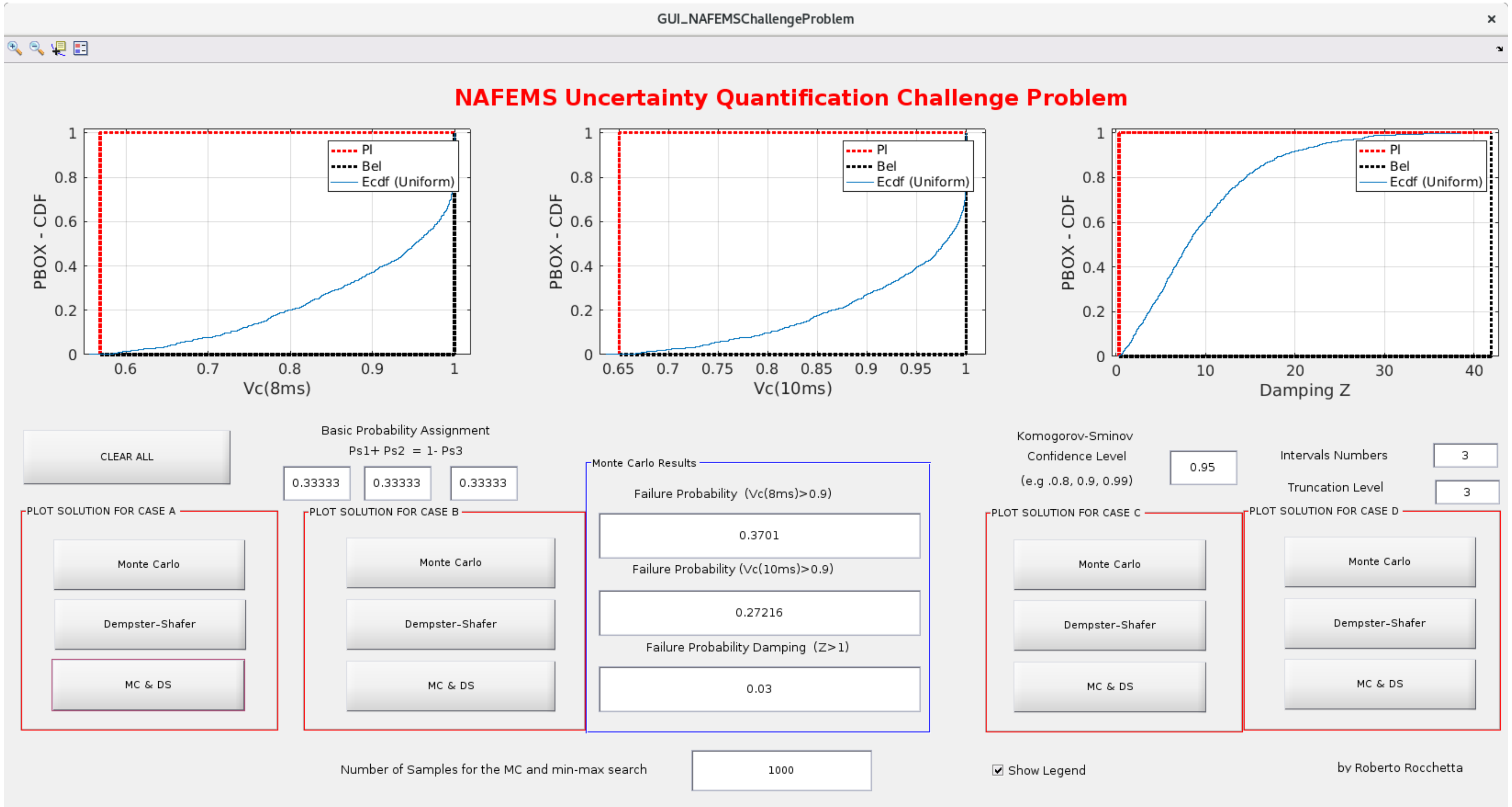


CASE-A

Expected Results



Hands-on of imprecise simulation in engineering

Outline

CASE-B

Multiple intervals

Information available:

- 9 intervals, 3 sources for each system parameter.
- Source 1 correspond to CASE-A.

CASE-B	R [Ω]	L [mH]	C [μF]
source 1	[40,1000]	[1,10]	[1,10]
source 2	[600,1200]	[10,100]	[1,10]
source 3	[10,1500]	[4,8]	[0.5,4]

CASE-B

Multiple intervals

Information available:

- 9 intervals, 3 sources for each system parameter.
- Source 1 correspond to CASE-A.

CASE-B	R [Ω]	L [mH]	C [μF]
source 1	[40,1000]	[1,10]	[1,10]
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Example of probabilistic approach

- By the maximum entropy principle assume 9 uniform PDFs ($R_1 \sim U_{r1}$, $R_2 \sim U_{r2}$, etc.);
- Propagate uncertainty with Monte Carlo and perform reliability analysis
- Failure probabilities computed for each source of information ($\hat{P}_{Vc10,1}$, $\hat{P}_{tr,1}$ $\hat{P}_{Z,1}$, etc.);

CASE-B

Double Monte Carlo

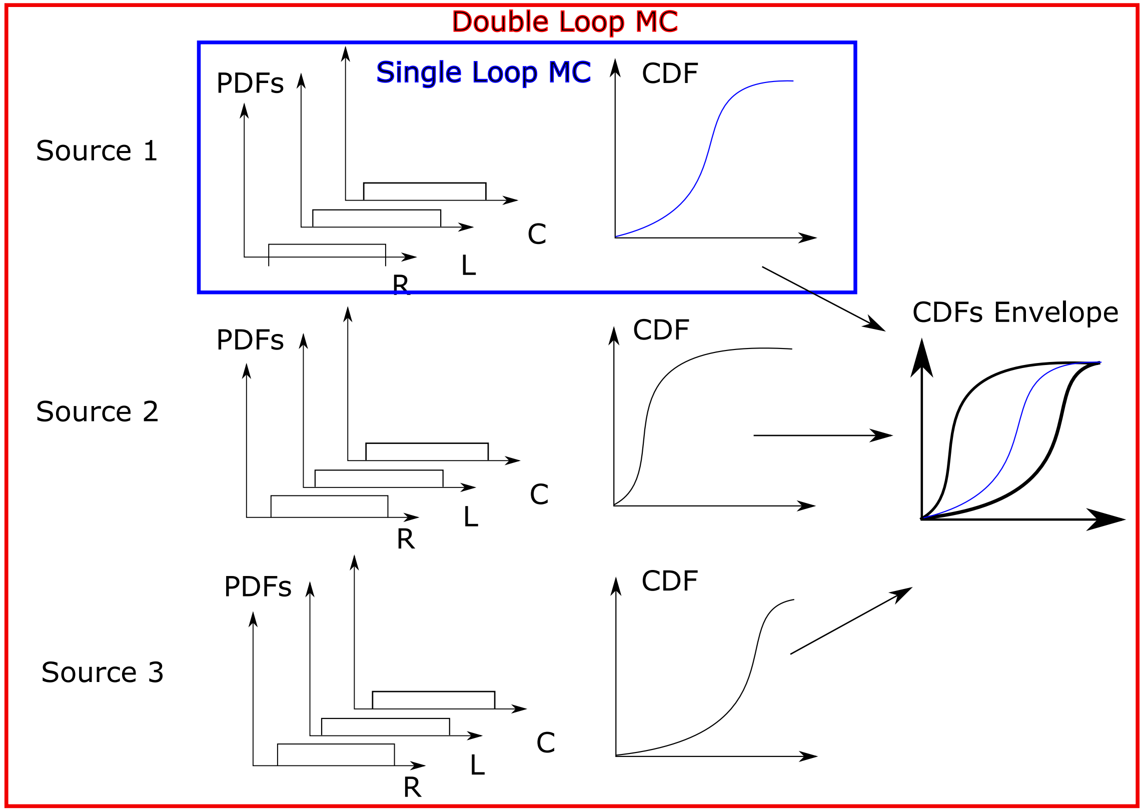

Figure 9: Comparision between single loop and double loop Monte Carlo

CASE-B

Multiple intervals

Information available:

- 9 intervals, 3 sources for each system parameter.
- Source 1 correspond to CASE-A.

CASE-B	R [Ω]	L [mH]	C [μF]
source 1	[40,1000]	[1,10]	[1,10]
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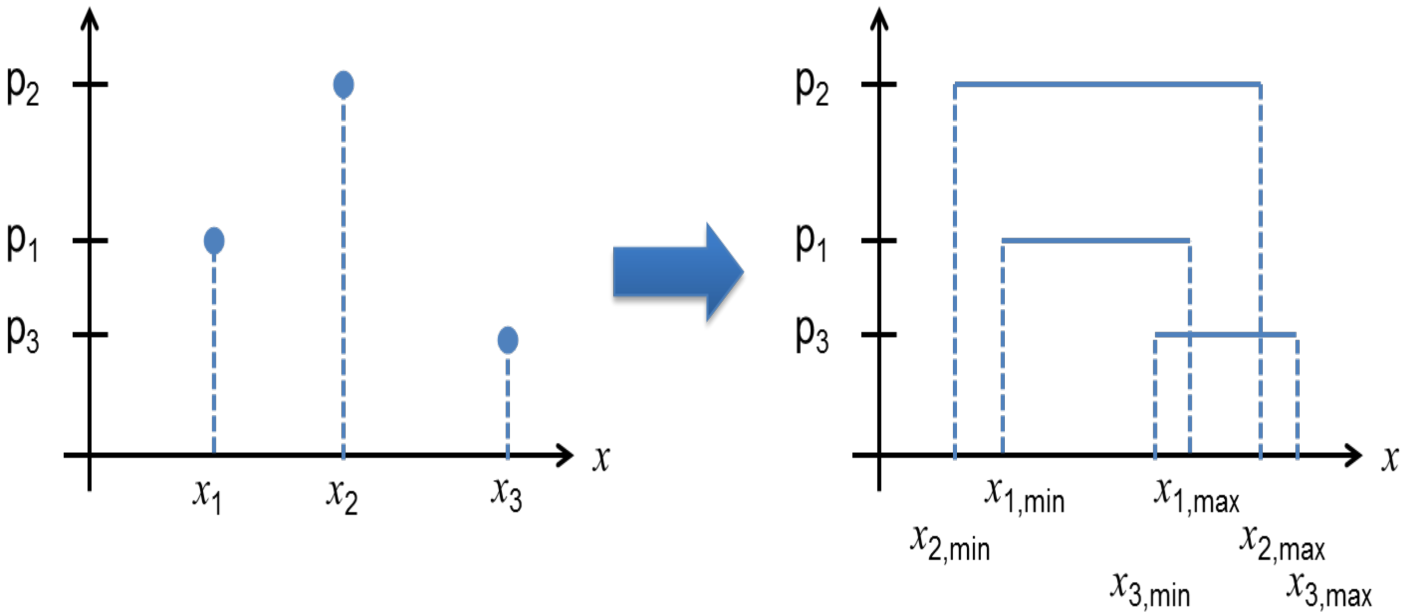
Dempster-Shafer structures

- Assign probability masses to the 3 sources (e.g. $m_{1,2,3} = \frac{1}{3}$)
- Combine focal elements (3^3) and compute min-max $V_c(10ms)$, t_r and Z and probability mass for each combination;
- $\hat{P}_{Vc10}$, $\hat{P}_{tr}$, $\hat{P}_Z$ are intervals;

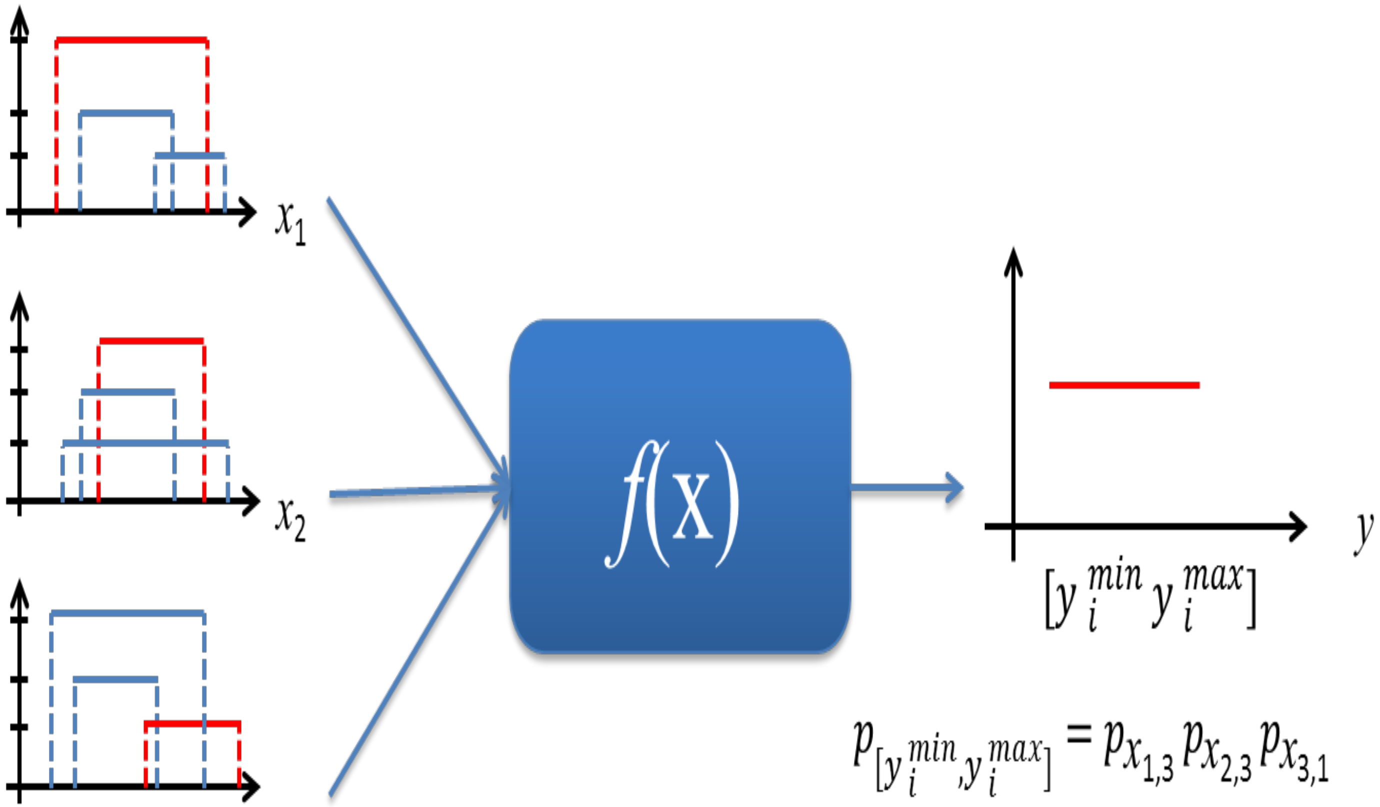
CASE-B

D-S structure



CASE-B

D-S structure



CASE-B

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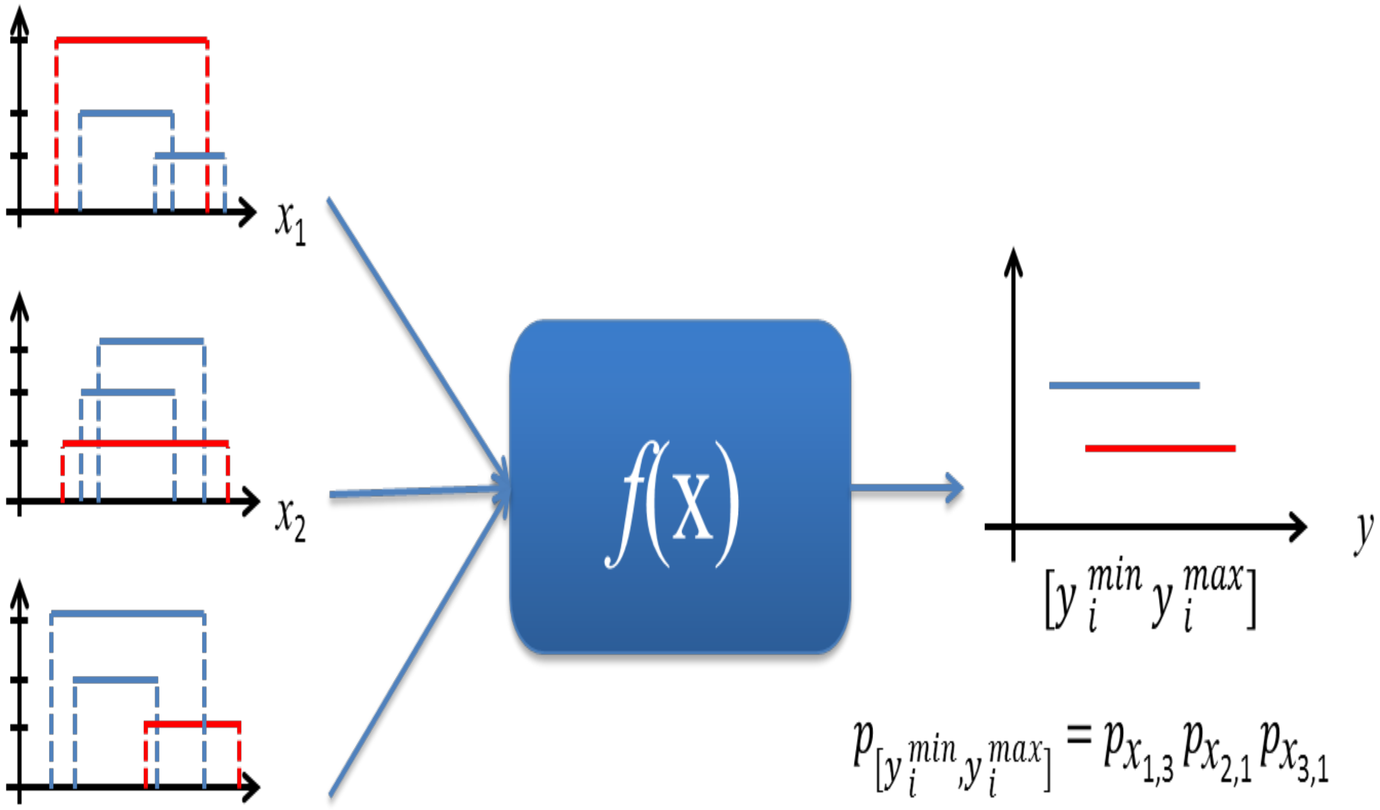
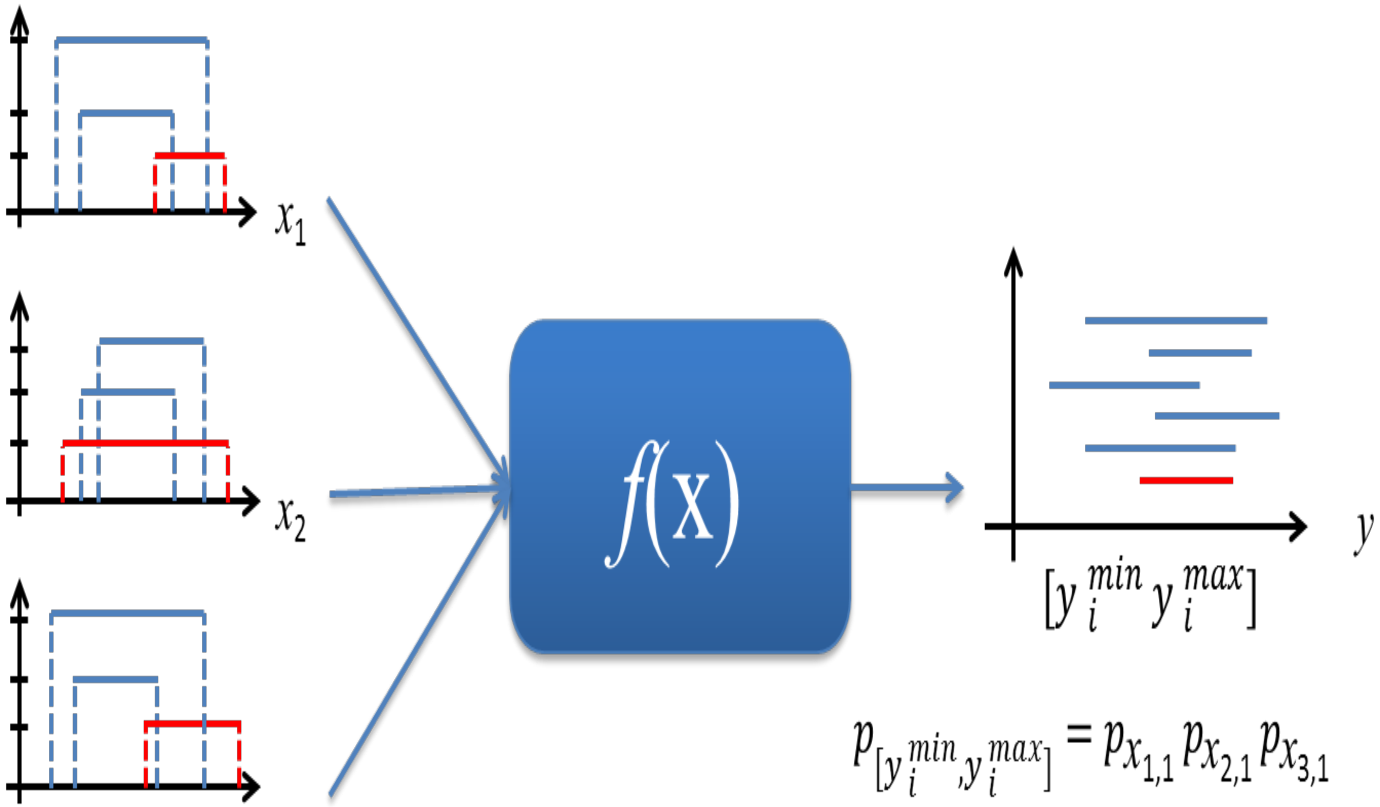


CASE-B

D-S structure



Transform DS to Pbox

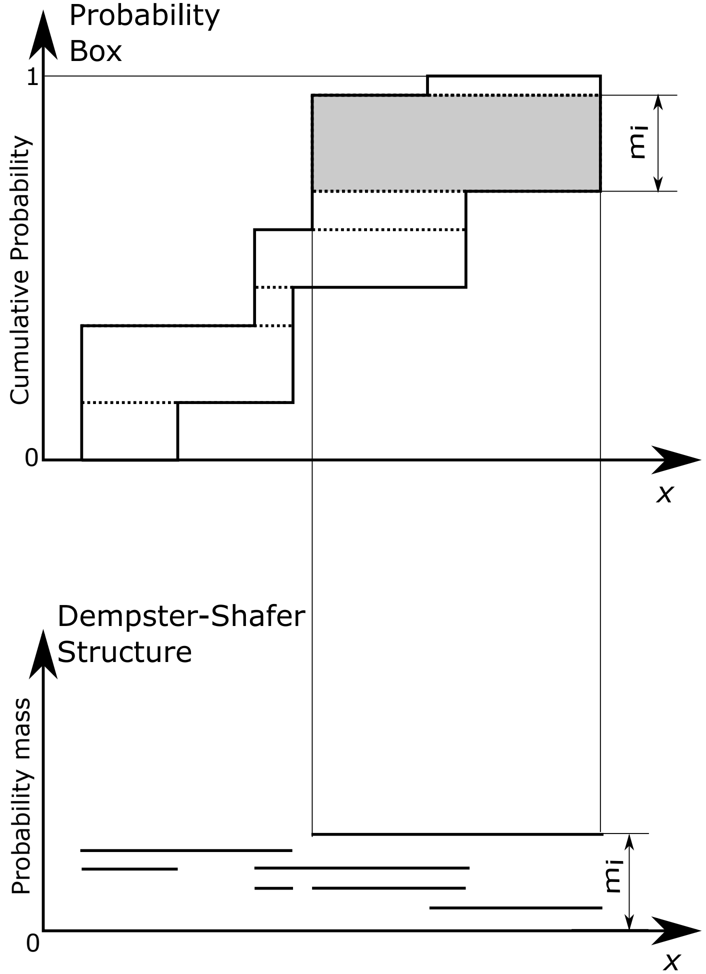
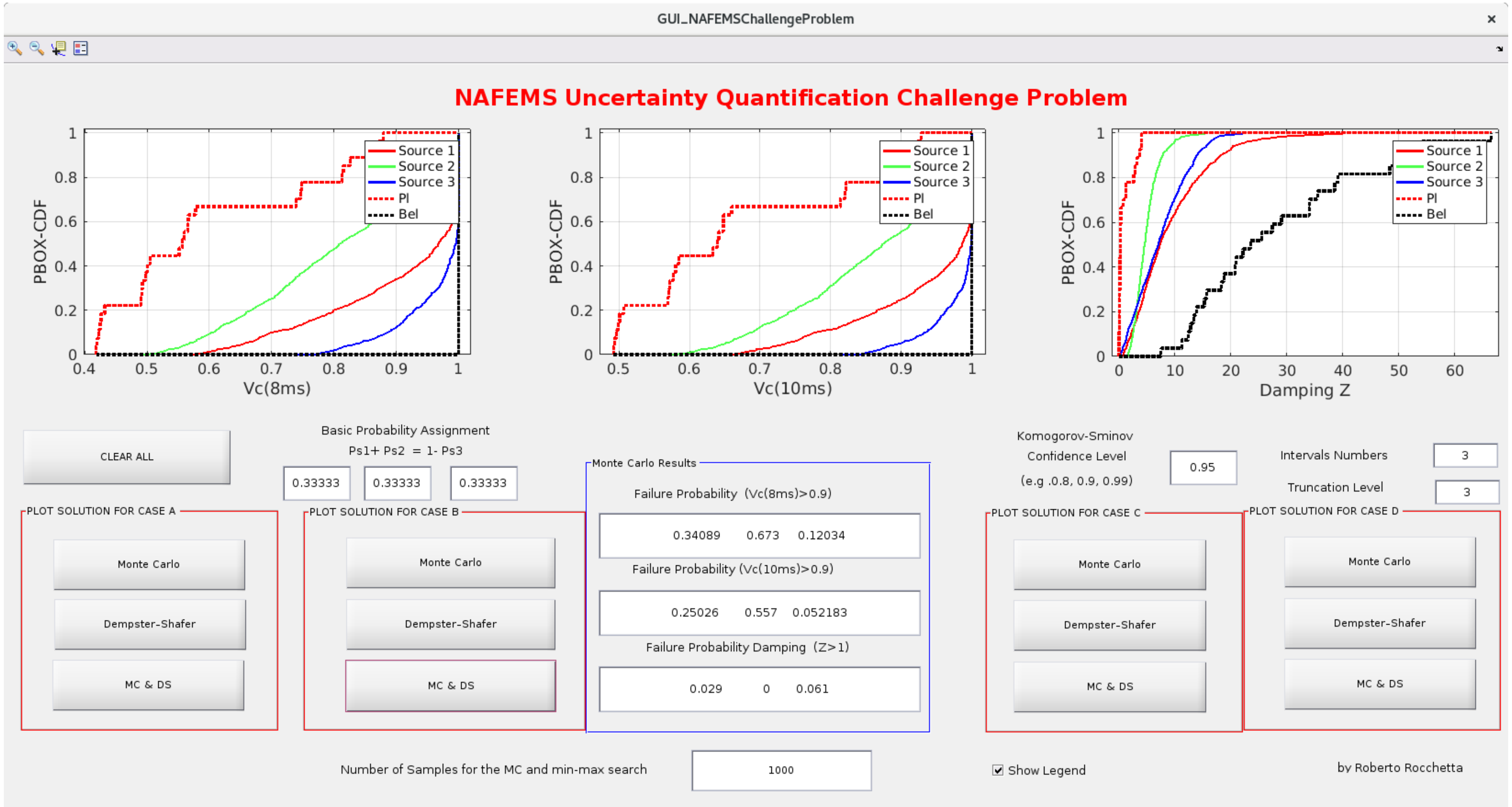


Figure 10: Transform a DS structure in a Pbox and vice-versa.

CASE-B

Expected Results



Hands-on of imprecise simulation in engineering

Outline

CASE-C

Sampled Points

Information available:

- 10 sampled values for each system parameter

Information:

CASE-C	R [Ω]	L [mH]	C [μF]
Sampled Data	861, 87, 430, 798, 219, 152, 64, 361, 224, 61	4.1, 8.8, 4.0, 7.6, 0.7, 3.9, 7.1, 5.9, 8.2, 5.1	9.0, 5.2, 3.8, 4.9, 2.9, 8.3, 7.7, 5.8, 10, 0.7

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Probabilistic approach

- PDF fitting for R,L and C (e.g. Kernel Density Estimation);
- Propagate uncertainty with MC and compute $\hat{P}_{Vc10}$, $\hat{P}_{tr}$ $\hat{P}_{Z}$;

CASE-C

Kernel Density Estimation

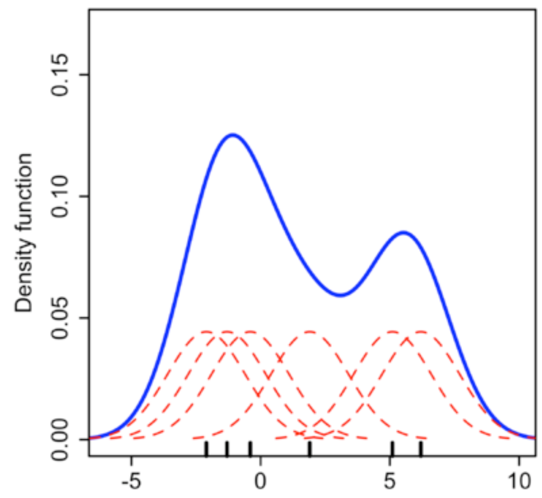
Hypothesis: 10 samples $x_1, x_2, \dots, x_{10}$ IID drawn from some distribution with an unknown density f.

$$\hat{f}_h(x) = \frac{1}{n}\sum_{i=1}^{n} K_h(x - x_i) = \frac{1}{nh}\sum_{i=1}^{n} K\Big(\frac{x - x_i}{h}\Big), \qquad (49)$$

where $K(\cdot)$ is the kernel, $h > 0$ is a smoothing parameter called the bandwidth.

For Gaussian basis functions used to approximate univariate data, *Silverman's rule of thumb*:

$$h = \left(\frac{4\hat{\sigma}^5}{3n}\right)^{\frac{1}{5}} \approx 1.06\hat{\sigma}n^{-1/5}, \qquad (50)$$



$\hat{\sigma}$: standard deviation of the samples

Silverman, B.W. (1998). Density Estimation for Statistics and Data Analysis. London: Chapman & Hall/CRC. p. 48. ISBN 0-412-24620-1.

CASE-C

Available information

Information available:

- 10 sampled values for each system parameter

Information:

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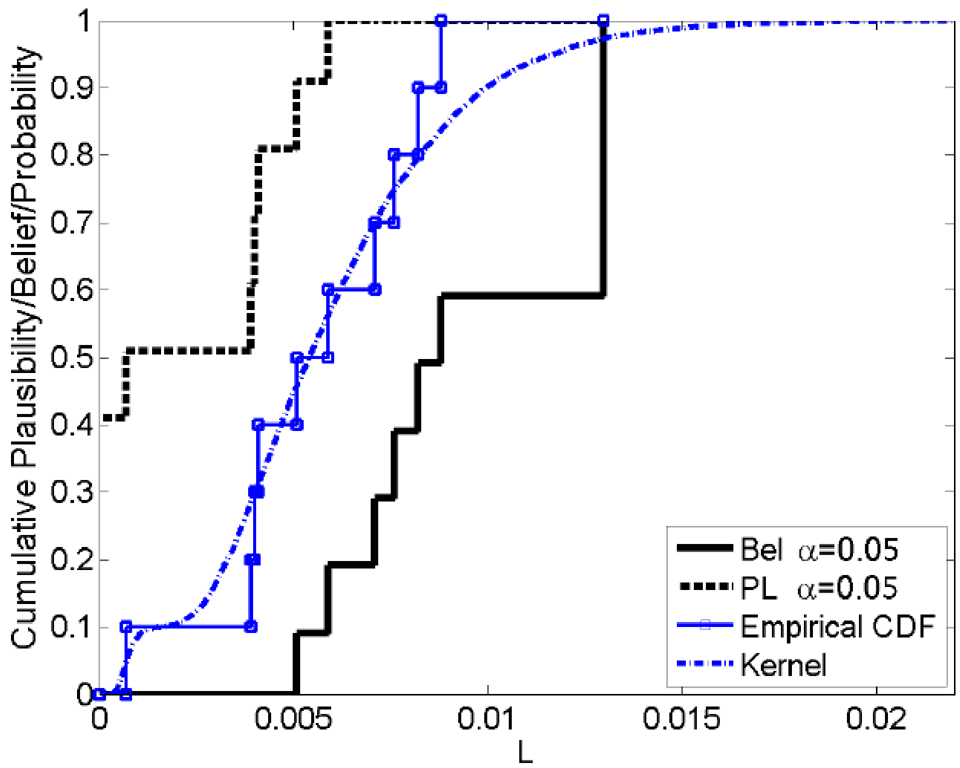
Imprecise probability

- Kolmogorov-Smirnov test (i.e. confidence bounds on the CDF and characterization using P-box);
- P-box propagation and compute $\hat{P}_{Vc10}$, $\hat{P}_{tr}$, $\hat{P}_{Z}$, which again are intervals;

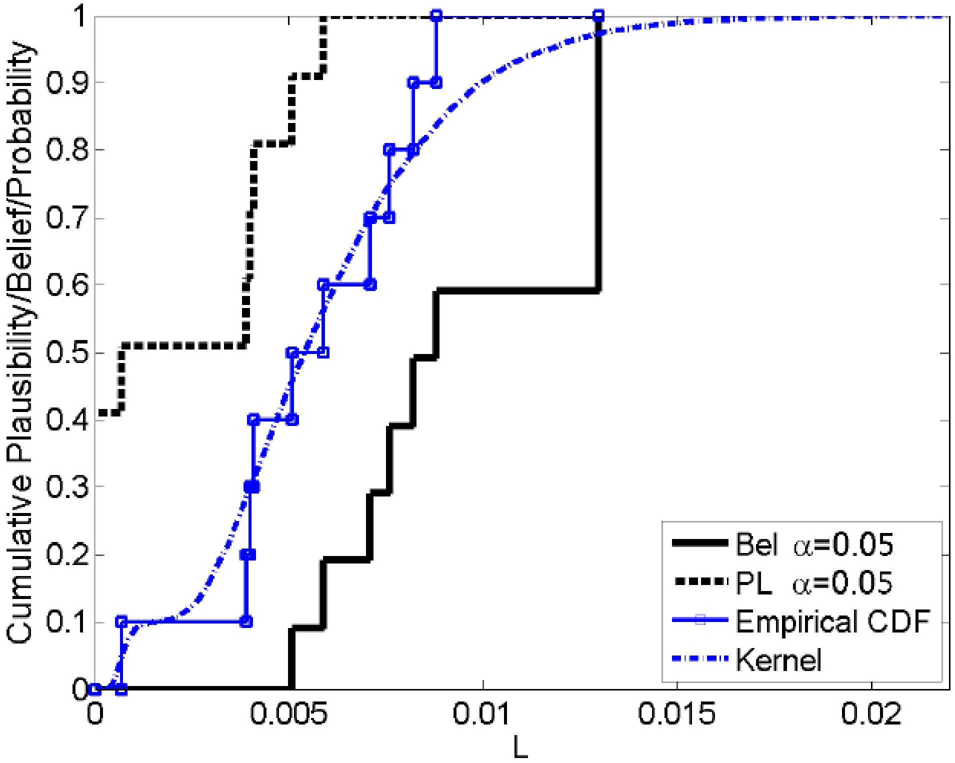
CASE-C

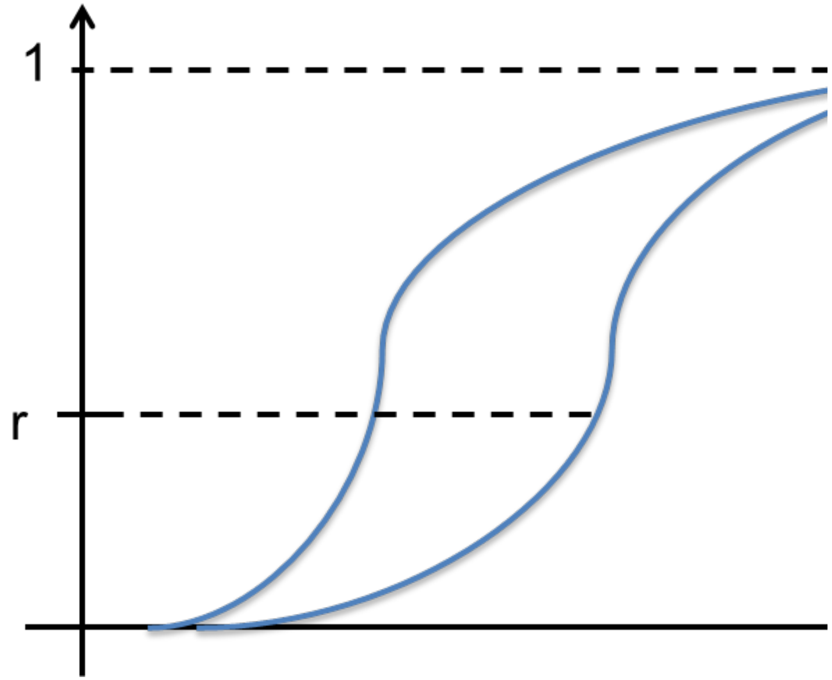
Probability Boxes



CASE-C

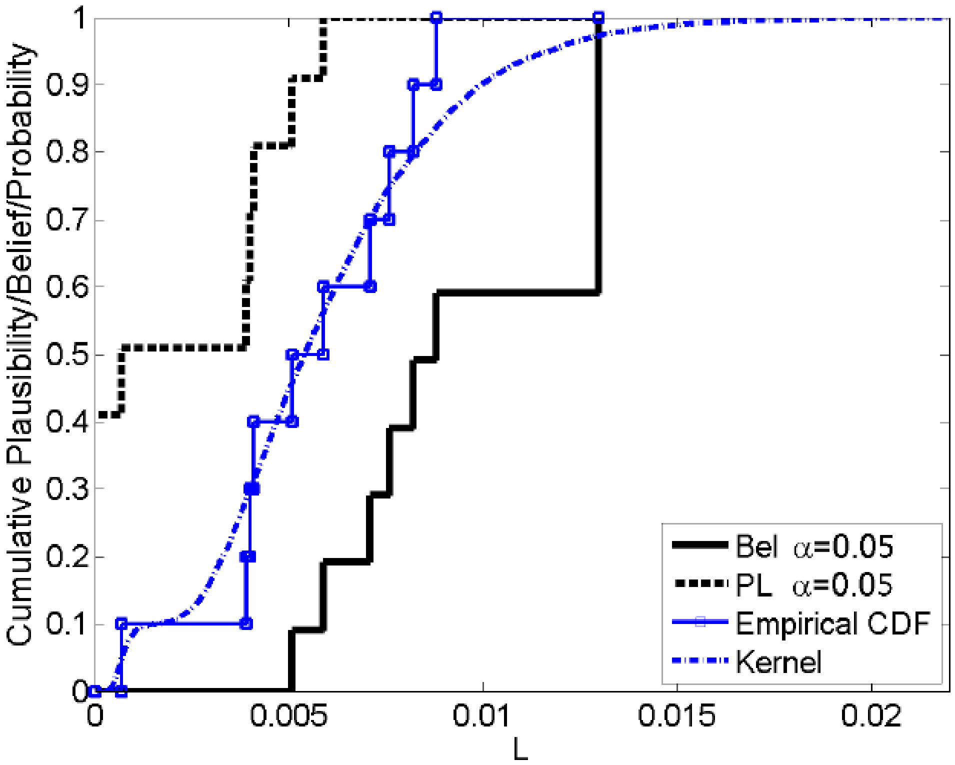
Probability Boxes

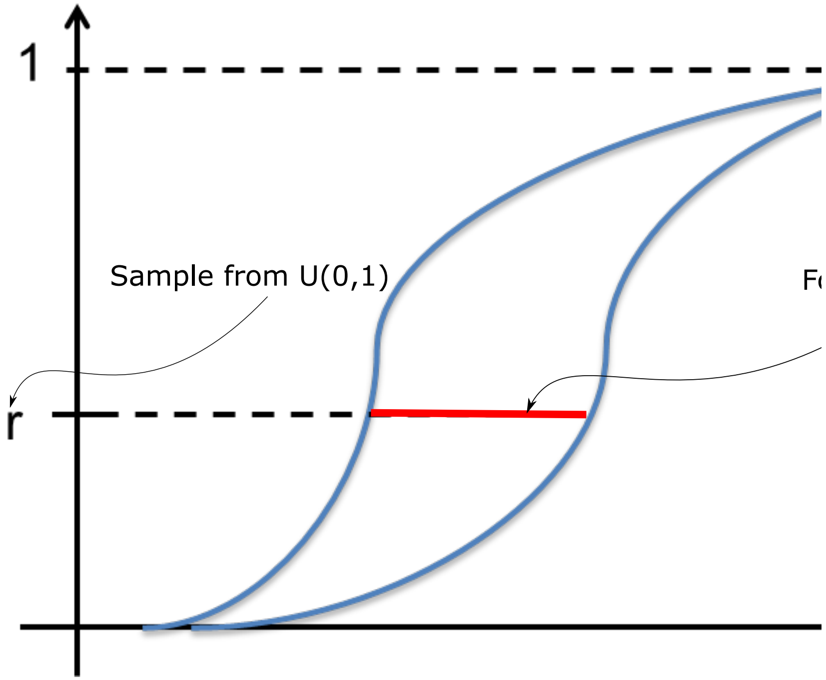




CASE-C

Probability Boxes





CASE-C

Bayesian Updating

$$P(\boldsymbol{\theta}|D, I) = \frac{P(D|\boldsymbol{\theta}, I)\, P(\boldsymbol{\theta}|I)}{P(D|I)} \tag{51}$$

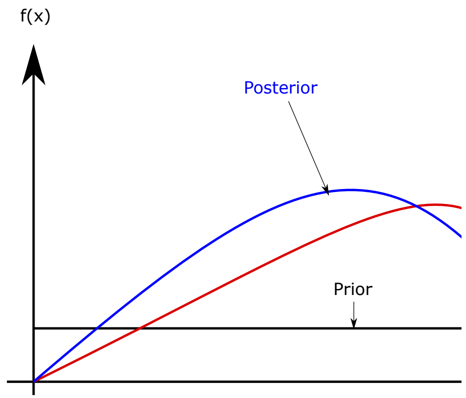
Bayesian Approach:

- Assume prior distribution (e.g. $P(\boldsymbol{\theta}|I)$ as resulting from CASE-A)
- Collect data and compute likelihood

$$P(D|\boldsymbol{\theta}, I) = \prod_{k=1}^{N_e} P(x_k^e; \boldsymbol{\theta}) = \sum_{k=1}^{N_e} \log(P(x_k^e; \boldsymbol{\theta}))$$

Compute Posterior $P(\boldsymbol{\theta}|D, I) \propto P(D|\boldsymbol{\theta}, I)\, P(\boldsymbol{\theta}|I)$ for instance

$$P(x_k^e; \boldsymbol{\theta}) \propto \exp\left(\frac{N}{\sum_{j=1}^{N}\left[f(\boldsymbol{\theta}, \omega_j) - f_k^e(\omega_j)\right]^2}\right)$$



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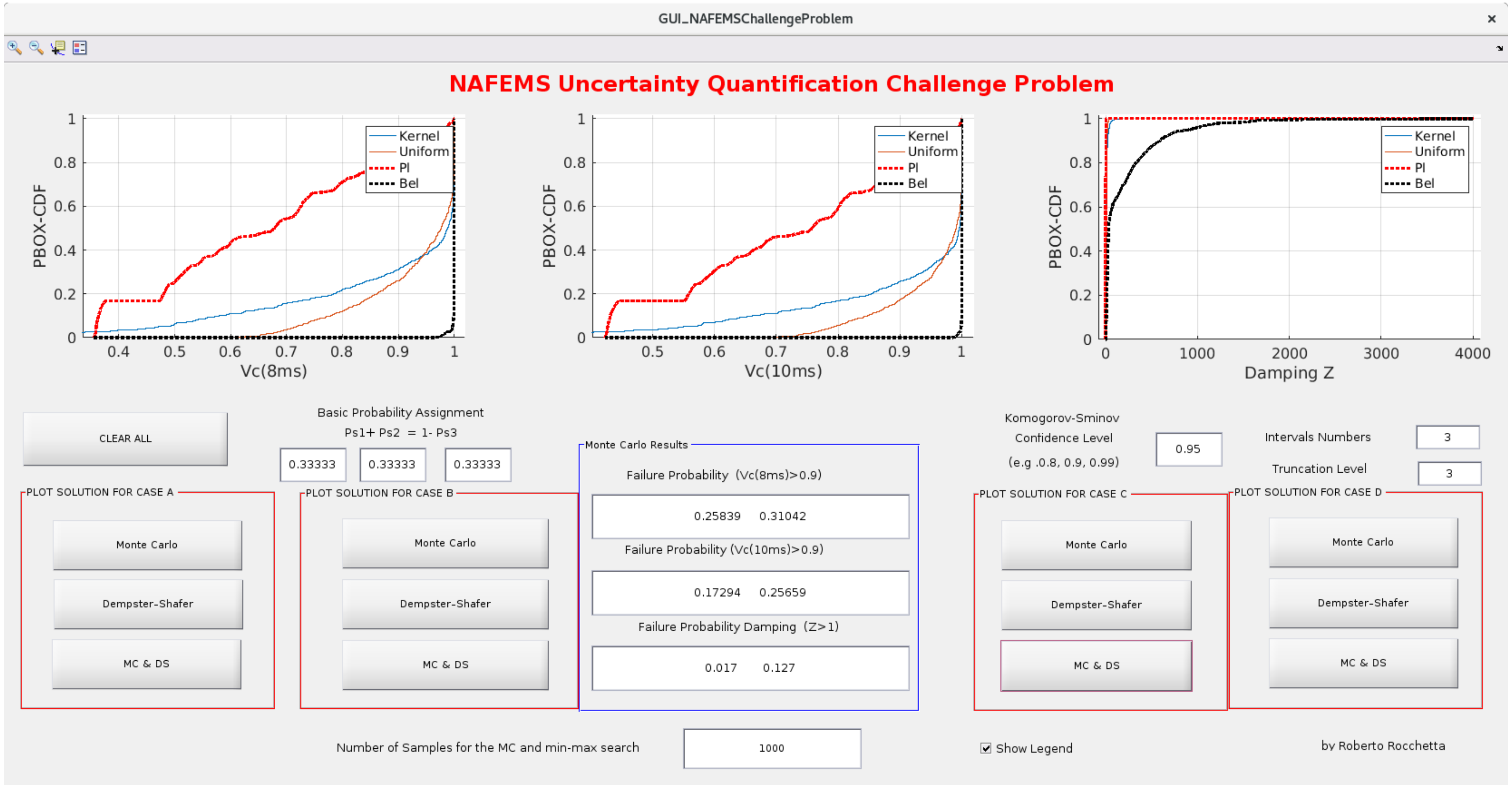
CASE-C

Expected Results



Outline

CASE-D

Incomplete data

Information available:

- Nominal value and unbounded intervals

CASE-D	R [Ω]	L [mH]	C [μF]
Interval	[40,R_{U1}]	[1,L_{U1}]	[C_{L1},10]
Other info	R_{U1} >650	L_{U1} >6	C_{L1} <7
Nominal Val.	650	6	7

Minimum bounds can be fixed using physical considerations (non-negativity), what about the upper bounds? What is the meaning of Nominal Value here? How can we use it?

CASE-D

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Minimum bounds can be fixed using physical considerations (non-negativity), what about the upper bounds? What is the meaning of Nominal Value here? How can we use it?

Probabilistic approach

- PDF fitting for R,L and C (Truncated Gaussian distribution? Uniform distribution? e.g. $R \sim U(40, R_n * k)$ where k is a user defined parameter
- Monte Carlo uncertainty propagation

CASE-D

Incomplete data

Information available:

- Nominal value and unbounded intervals

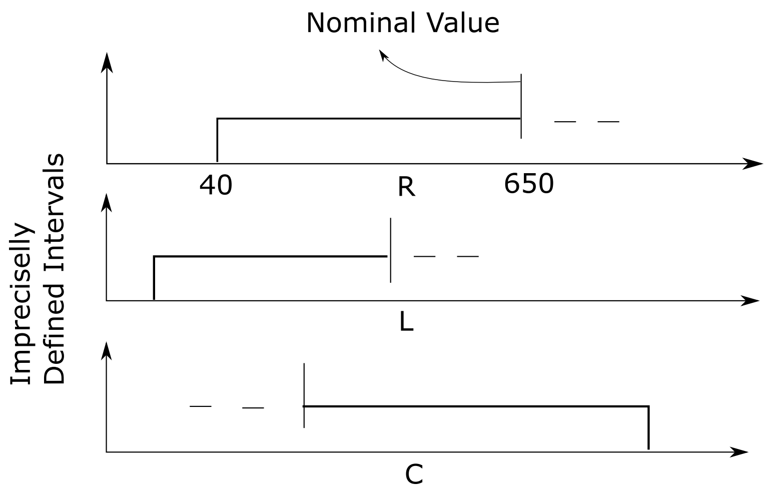
CASE-D	R [Ω]	L [mH]	C [μF]
Interval	[40,R_{U1}]	[1,L_{U1}]	[C_{L1},10]
Other info	R_{U1} >650	L_{U1} >6	C_{L1} <7
Nominal Val.	650	6	7

Imprecise probability

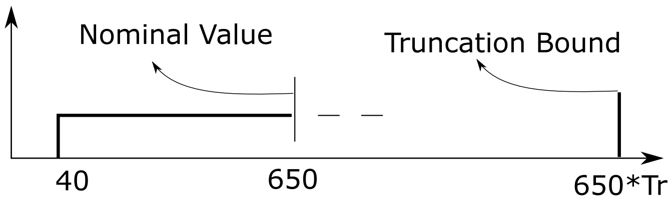
- D-S propagation? e.g. propagate 3-dimensional focal elements $\{[40, \infty]\ [0.006, \infty]\ [-\infty, 0.0000001]\}$;

CASE-D

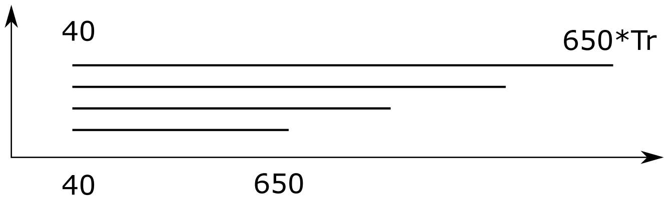
Incomplete data



Explore how the imprecisely defined bound influence the results

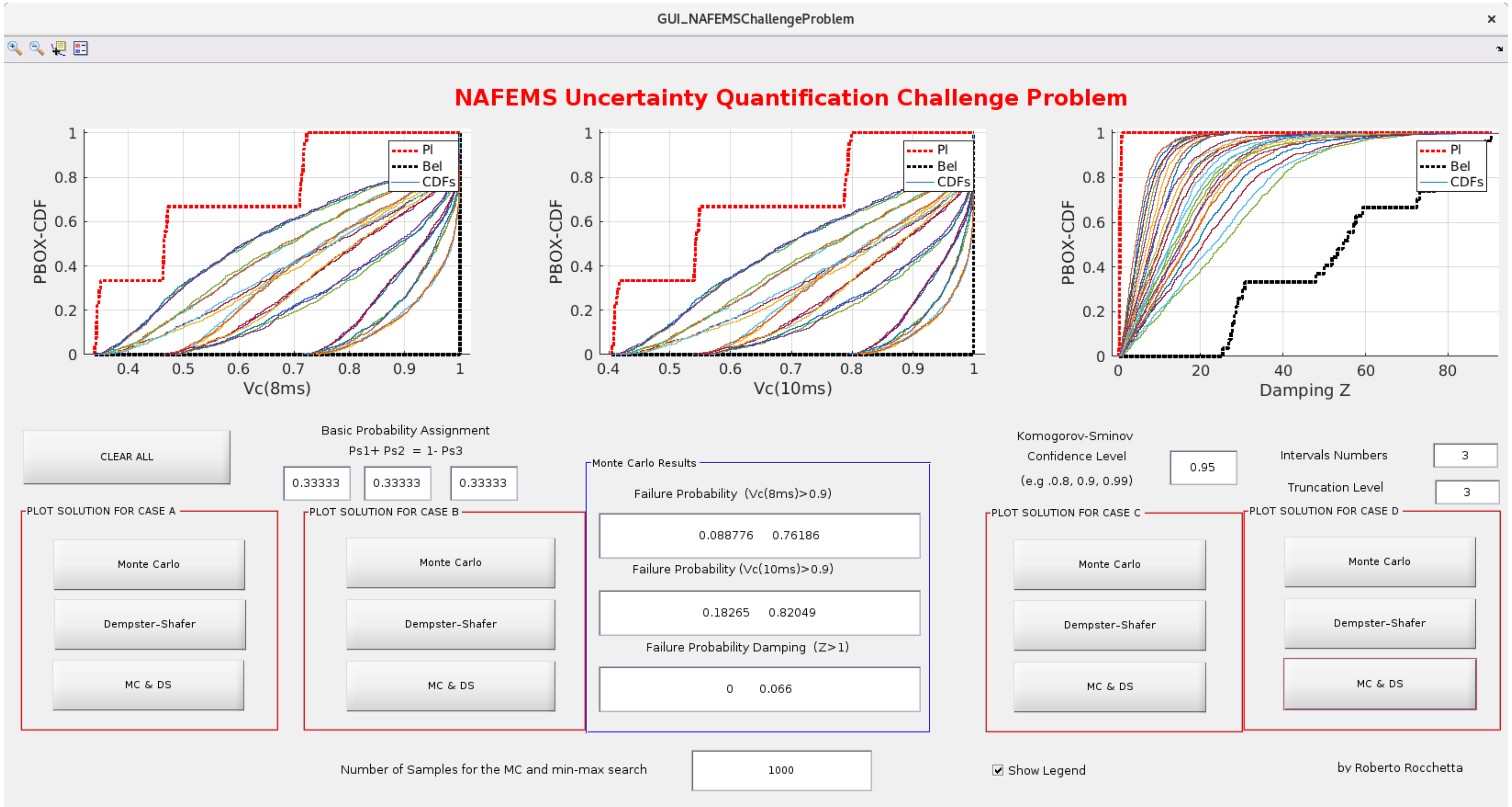


Assume a truncation bound and analyse a number of possible intervals (e.g. 4 in figure)



CASE-D

Expected Results



CASE-E

Combine all available information

- Consider all the sources of information (CASE-A,CASE-B,CASE-C,CASE-D)
- Sampled values, nominal value and bounded and unbounded intervals

Tasks

- Can we combine these sources of information?
- What is the effect of the reliability analysis?

Hands-on of imprecise simulation in engineering

Outline

NAFEMS UQ CHALLENGE PROBLEM

Combine all available information

Quality of the information in each case?

- Check the output intervals

Propagation of uncertainty

Some final considerations:

- Information of case A and D seems to have the lowest quality (P_f interval about [0 1])
- CASE-C has the higher quality (narrower bounds on the output)
- Monte Carlo P_f lay within the bounds obtained by DS and P-box approaches